

# 2

# Pythagoras Theorem



### Let's study.

- Pythagorean triplet
- Theorem of geometric mean
- Application of Pythagoras theorem
- Similarity and right angled triangles
- Pythagoras theorem
- Apollonius theorem



### Let's recall.

#### Pythagoras theorem :

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of remaining two sides.

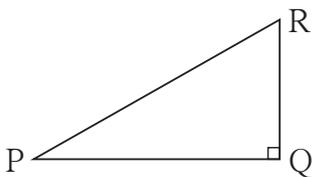


Fig. 2.1

In  $\Delta PQR \angle PQR = 90^\circ$

$$l(PR)^2 = l(PQ)^2 + l(QR)^2$$

We will write this as,

$$PR^2 = PQ^2 + QR^2$$

The lengths PQ, QR and PR of  $\Delta PQR$  can also be shown by letters r, p and q. With this convention, referring to figure 2.1, Pythagoras theorem can also be stated as  $q^2 = p^2 + r^2$ .

#### Pythagorean Triplet :

In a triplet of natural numbers, if the square of the largest number is equal to the sum of the squares of the remaining two numbers then the triplet is called Pythagorean triplet.

For Example: In the triplet ( 11, 60, 61 ) ,

$$11^2 = 121, \quad 60^2 = 3600, \quad 61^2 = 3721 \quad \text{and} \quad 121 + 3600 = 3721$$

The square of the largest number is equal to the sum of the squares of the other two numbers.

$\therefore$  11, 60, 61 is a Pythagorean triplet.

Verify that (3, 4, 5), (5, 12, 13), (8, 15, 17), (24, 25, 7) are Pythagorean triplets.

Numbers in Pythagorean triplet can be written in any order.







## Pythagoras Theorem

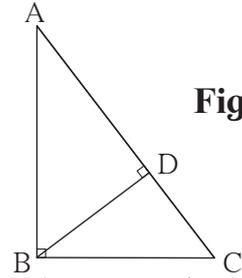
**In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of remaining two sides.**

**Given :** In  $\Delta ABC$ ,  $\angle ABC = 90^\circ$

**To prove :**  $AC^2 = AB^2 + BC^2$

**Construction :** Draw perpendicular seg BD on side AC.

A-D-C.



**Fig. 2.7**

**Proof :** In right angled  $\Delta ABC$ , seg  $BD \perp$  hypotenuse  $AC$  ..... (construction)

$\therefore \Delta ABC \sim \Delta ADB \sim \Delta BDC$  ..... (similarity of right angled triangles)

$\Delta ABC \sim \Delta ADB$

$$\frac{AB}{AD} = \frac{BC}{DB} = \frac{AC}{AB} \text{ - corresponding sides}$$

$$\frac{AB}{AD} = \frac{AC}{AB}$$

$$AB^2 = AD \times AC \text{ ..... (I)}$$

Similarly,  $\Delta ABC \sim \Delta BDC$

$$\frac{AB}{BD} = \frac{BC}{DC} = \frac{AC}{BC} \text{ -corresponding sides}$$

$$\frac{BC}{DC} = \frac{AC}{BC}$$

$$BC^2 = DC \times AC \text{ ..... (II)}$$

Adding (I) and (II)

$$\begin{aligned} AB^2 + BC^2 &= AD \times AC + DC \times AC \\ &= AC (AD + DC) \\ &= AC \times AC \text{ ..... (A-D-C)} \end{aligned}$$

$$\therefore AB^2 + BC^2 = AC^2$$

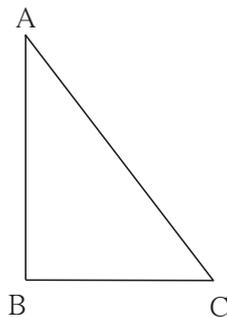
$$\therefore AC^2 = AB^2 + BC^2$$

## Converse of Pythagoras theorem

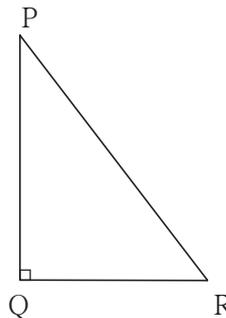
**In a triangle if the square of one side is equal to the sum of the squares of the remaining two sides, then the triangle is a right angled triangle.**

**Given :** In  $\Delta ABC$ ,  $AC^2 = AB^2 + BC^2$

**To prove :**  $\angle ABC = 90^\circ$



**Fig. 2.8**



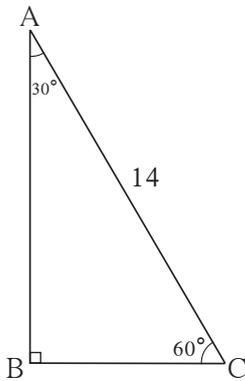
**Fig. 2.9**



**Solved Examples**

**Ex. (1)** See fig 2.11. In  $\triangle ABC$ ,  $\angle B = 90^\circ$ ,  $\angle A = 30^\circ$ ,  $AC = 14$ , then find  $AB$  and  $BC$

**Solution :**



**Fig. 2.11**

In  $\triangle ABC$ ,

$$\angle B = 90^\circ, \angle A = 30^\circ, \therefore \angle C = 60^\circ$$

By  $30^\circ - 60^\circ - 90^\circ$  theorem,

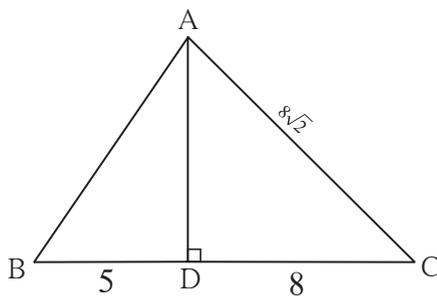
$$BC = \frac{1}{2} \times AC \qquad \qquad \qquad AB = \frac{\sqrt{3}}{2} \times AC$$

$$BC = \frac{1}{2} \times 14 \qquad \qquad \qquad AB = \frac{\sqrt{3}}{2} \times 14$$

$$BC = 7 \qquad \qquad \qquad AB = 7\sqrt{3}$$

**Ex. (2)** See fig 2.12, In  $\triangle ABC$ , seg  $AD \perp$  seg  $BC$ ,  $\angle C = 45^\circ$ ,  $BD = 5$  and  $AC = 8\sqrt{2}$  then find  $AD$  and  $BC$ .

**Solution :** In  $\triangle ADC$



**Fig. 2.12**

$$\angle ADC = 90^\circ, \angle C = 45^\circ, \therefore \angle DAC = 45^\circ$$

$$AD = DC = \frac{1}{\sqrt{2}} \times 8\sqrt{2} \dots \text{by } 45^\circ - 45^\circ - 90^\circ \text{ theorem}$$

$$DC = 8 \quad \therefore \quad AD = 8$$

$$BC = BD + DC$$

$$= 5 + 8$$

$$BC = 13$$

**Ex. (3)** In fig 2.13,  $\angle PQR = 90^\circ$ , seg  $QN \perp$  seg  $PR$ ,  $PN = 9$ ,  $NR = 16$ . Find  $QN$ .

**Solution :** In  $\triangle PQR$ , seg  $QN \perp$  seg  $PR$

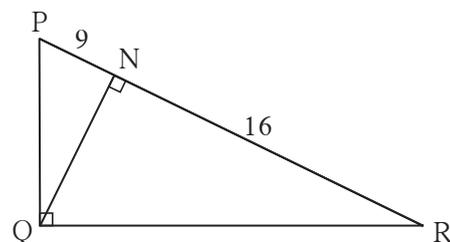
$$NQ^2 = PN \times NR \dots \text{theorem of geometric mean}$$

$$\therefore NQ = \sqrt{PN \times NR}$$

$$= \sqrt{9 \times 16}$$

$$= 3 \times 4$$

$$= 12$$



**Fig. 2.13**









Let's learn.

### Application of Pythagoras theorem

In Pythagoras theorem, the relation between hypotenuse and sides making right angle i.e. the relation between side opposite to right angle and the remaining two sides is given.

In a triangle, relation between the side opposite to acute angle and remaining two sides and relation of the side opposite to obtuse angle with remaining two sides can be determined with the help of Pythagoras theorem. Study these relations from the following examples.

**Ex. (1)** In  $\Delta ABC$ ,  $\angle C$  is an acute angle, seg  $AD \perp$  seg  $BC$ . Prove that:

$$AB^2 = BC^2 + AC^2 - 2BC \times DC$$

In the given figure let  $AB = c$ ,  $AC = b$ ,  $AD = p$ ,  $BC = a$ ,  $DC = x$ ,

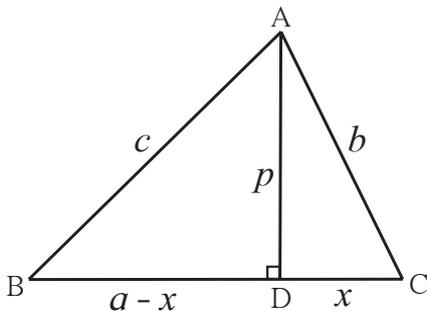


Fig. 2.23

$$\therefore BD = a - x$$

In  $\Delta ADB$ , by Pythagoras theorem

$$c^2 = (a-x)^2 + \square$$

$$c^2 = a^2 - 2ax + x^2 + \square \dots\dots\dots (I)$$

In  $\Delta ADC$ , by Pythagoras theorem

$$b^2 = p^2 + \square$$

$$p^2 = b^2 - \square \dots\dots\dots (II)$$

Substituting value of  $p^2$  from (II) in (I),

$$c^2 = a^2 - 2ax + x^2 + b^2 - x^2$$

$$\therefore c^2 = a^2 + b^2 - 2ax$$

$$\therefore AB^2 = BC^2 + AC^2 - 2BC \times DC$$

**Ex. (2)** In  $\Delta ABC$ ,  $\angle ACB$  is obtuse angle, seg  $AD \perp$  seg  $BC$ . Prove that:

$$AB^2 = BC^2 + AC^2 + 2BC \times CD$$

In the figure seg  $AD \perp$  seg  $BC$

Let  $AD = p$ ,  $AC = b$ ,  $AB = c$ ,

$BC = a$  and  $DC = x$ .

$$DB = a + x$$

In  $\Delta ADB$ , by Pythagoras theorem,

$$c^2 = (a + x)^2 + p^2$$

$$c^2 = a^2 + 2ax + x^2 + p^2 \dots\dots\dots (I)$$

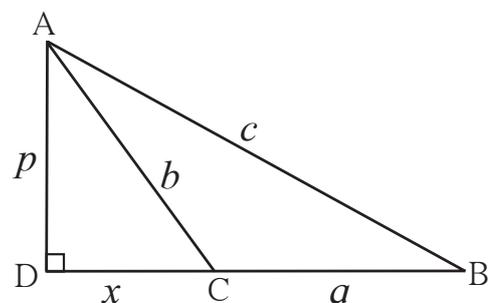


Fig. 2.24

Similarly, in  $\Delta ADC$

$$b^2 = x^2 + p^2$$

$$\therefore p^2 = b^2 - x^2 \quad \dots\dots\dots (II)$$

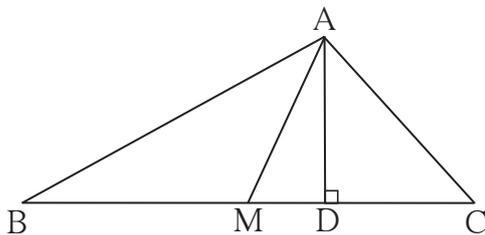
$\therefore$  substituting the value of  $p^2$  from (II) in (I)

$$\therefore c^2 = a^2 + 2ax + b^2$$

$$\therefore AB^2 = BC^2 + AC^2 + 2BC \times CD$$

**Apollonius theorem**

In  $\Delta ABC$ , if M is the midpoint of side BC, then  $AB^2 + AC^2 = 2AM^2 + 2BM^2$



**Fig. 2.25**

**Given** : In  $\Delta ABC$ , M is the midpoint of side BC.

**To prove** :  $AB^2 + AC^2 = 2AM^2 + 2BM^2$

**Construction**: Draw seg  $AD \perp$  seg BC

**Proof** : If seg AM is not perpendicular to seg BC then out of  $\angle AMB$  and  $\angle AMC$  one is obtuse angle and the other is acute angle

In the figure,  $\angle AMB$  is obtuse angle and  $\angle AMC$  is acute angle.

From examples (1) and (2) above,

$$AB^2 = AM^2 + MB^2 + 2BM \times MD \quad \dots\dots (I)$$

$$\text{and } AC^2 = AM^2 + MC^2 - 2MC \times MD$$

$$\therefore AC^2 = AM^2 + MB^2 - 2BM \times MD \quad (\because BM = MC) \quad \dots\dots\dots (II)$$

$\therefore$  adding (I) and (II)

$$AB^2 + AC^2 = 2AM^2 + 2BM^2$$

Write the proof yourself if seg  $AM \perp$  seg BC.

From this example we can see the relation among the sides and medians of a triangle.

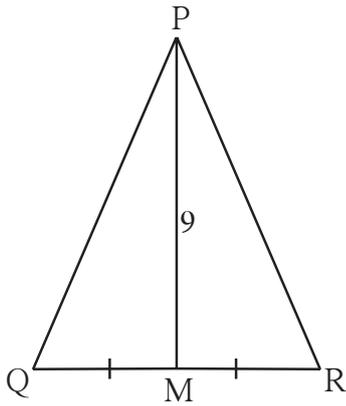
This is known as Apollonius theorem.

**Solved Examples**

**Ex. (1)** In the figure 2.26, seg PM is a median of  $\Delta PQR$ .  $PM = 9$  and  $PQ^2 + PR^2 = 290$ , then find QR.

**Solution** : In  $\Delta PQR$ , seg PM is a median.

M is the midpoint of seg QR.



**Fig. 2.26**

$$QM = MR = \frac{1}{2} QR$$

$$PQ^2 + PR^2 = 2PM^2 + 2QM^2 \text{ (by Apollonius theorem)}$$

$$290 = 2 \times 9^2 + 2QM^2$$

$$290 = 2 \times 81 + 2QM^2$$

$$290 = 162 + 2QM^2$$

$$2QM^2 = 290 - 162$$

$$2QM^2 = 128$$

$$QM^2 = 64$$

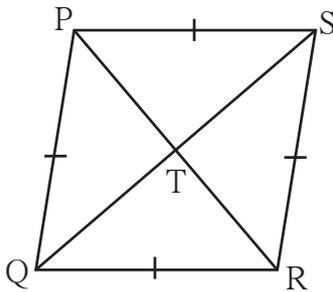
$$QM = 8$$

$$\therefore QR = 2 \times QM$$

$$= 2 \times 8$$

$$= 16$$

**Ex. (2)** Prove that, the sum of the squares of the diagonals of a rhombus is equal to the sum of the squares of the sides.



**Fig. 2.27**

**Given :** □ PQRS is a rhombus. Diagonals PR and SQ intersect each other at point T

**To prove :**  $PS^2 + SR^2 + QR^2 + PQ^2 = PR^2 + QS^2$

**Proof :** Diagonals of a rhombus bisect each other .

$\therefore$  by Apollonius' theorem,

$$PQ^2 + PS^2 = 2PT^2 + 2QT^2 \dots\dots\dots (I)$$

$$QR^2 + SR^2 = 2RT^2 + 2QT^2 \dots\dots\dots (II)$$

$\therefore$  adding (I) and (II) ,

$$PQ^2 + PS^2 + QR^2 + SR^2 = 2(PT^2 + RT^2) + 4QT^2$$

$$= 2(PT^2 + PT^2) + 4QT^2 \dots\dots\dots (RT = PT)$$

$$= 4PT^2 + 4QT^2$$

$$= (2PT)^2 + (2QT)^2$$

$$= PR^2 + QS^2$$

(The above proof can be written using Pythagoras theorem also.)

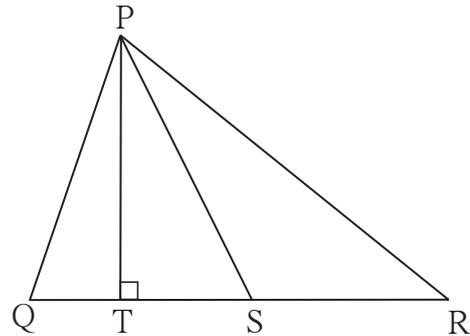
**Practice set 2.2**

1. In  $\Delta PQR$ , point S is the midpoint of side QR. If  $PQ = 11, PR = 17, PS = 13$ , find QR.
2. In  $\Delta ABC$ ,  $AB = 10, AC = 7, BC = 9$  then find the length of the median drawn from point C to side AB
3. In the figure 2.28 seg PS is the median of  $\Delta PQR$  and  $PT \perp QR$ .

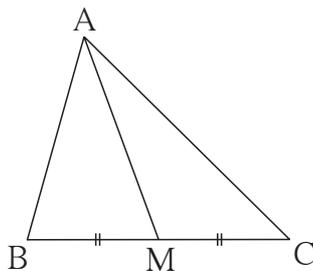
Prove that,

(1)  $PR^2 = PS^2 + QR \times ST + \left(\frac{QR}{2}\right)^2$

ii)  $PQ^2 = PS^2 - QR \times ST + \left(\frac{QR}{2}\right)^2$



**Fig. 2.28**



**Fig. 2.29**

4. In  $\Delta ABC$ , point M is the midpoint of side BC.

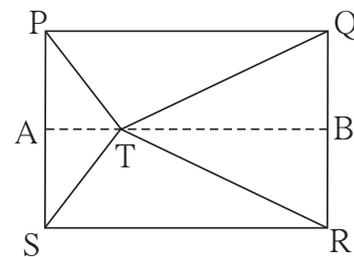
If,  $AB^2 + AC^2 = 290 \text{ cm}^2$ ,

$AM = 8 \text{ cm}$ , find BC.

- 5\*. In figure 2.30, point T is in the interior of rectangle PQRS,

Prove that,  $TS^2 + TQ^2 = TP^2 + TR^2$

(As shown in the figure, draw seg AB  $\parallel$  side SR and A-T-B)



**Fig. 2.30**

**Problem set 2**

1. Some questions and their alternative answers are given. Select the correct alternative.

(1) Out of the following which is the Pythagorean triplet?

- (A) (1, 5, 10)    (B) (3, 4, 5)    (C) (2, 2, 2)    (D) (5, 5, 2)

(2) In a right angled triangle, if sum of the squares of the sides making right angle is 169 then what is the length of the hypotenuse?

- (A) 15    (B) 13    (C) 5    (D) 12





15. In a trapezium ABCD,  
 seg AB  $\parallel$  seg DC  
 seg BD  $\perp$  seg AD,  
 seg AC  $\perp$  seg BC,  
 If AD = 15, BC = 15  
 and AB = 25. Find A( $\square$  ABCD)

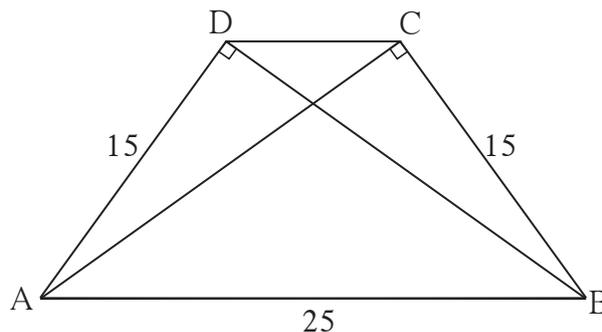


Fig. 2.34

- 16\*. In the figure 2.35,  $\Delta$  PQR is an equilateral triangle. Point S is on seg QR such that  $QS = \frac{1}{3} QR$ .  
 Prove that :  $9 PS^2 = 7 PQ^2$

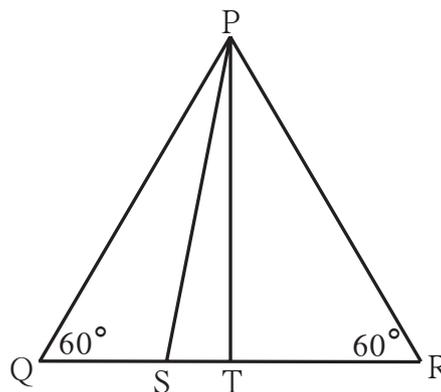
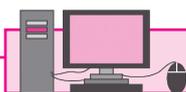


Fig. 2.35

- 17\*. Seg PM is a median of  $\Delta$  PQR. If PQ = 40, PR = 42 and PM = 29, find QR.  
 18. Seg AM is a median of  $\Delta$  ABC. If AB = 22, AC = 34, BC = 24, find AM



ICT Tools or Links

Obtain information on ‘the life of Pythagoras’ from the internet. Prepare a slide show.

