

Fig. 3.2

Activity II : In the adjoining figure, seg QR is a chord of the circle with centre O. P is the midpoint of the chord QR. If $QR = 24$, $OP = 10$, find radius of the circle.

To find solution of the problem, write the theorems that are useful.

- (1) _____
 (2) _____

Using these theorems solve the problems.

Activity III : In the adjoining figure, M is the centre of the circle and seg AB is a diameter.
 seg $MS \perp$ chord AD
 seg $MT \perp$ chord AC
 $\angle DAB \cong \angle CAB$.

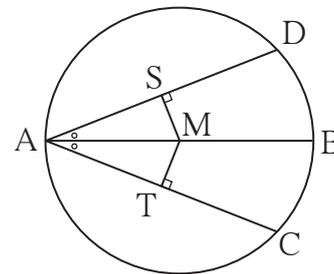


Fig. 3.3

Prove that : chord $AD \cong$ chord AC.

To solve this problem which of the following theorems will you use ?

- (1) The chords which are equidistant from the centre are equal in length.
 (2) Congruent chords of a circle are equidistant from the centre.

Which of the following tests of congruence of triangles will be useful?

- (1) SAS, (2) ASA, (3) SSS, (4) AAS, (5) hypotenuse-side test.

Using appropriate test and theorem write the proof of the above example.



Circles passing through one, two, three points

In the adjoining figure, point A lies in a plane. All the three circles with centres P, Q, R pass through point A. How many more such circles may pass through point A?

If your answer is many or innumerable, it is correct.

Infinite number of circles pass through a point.

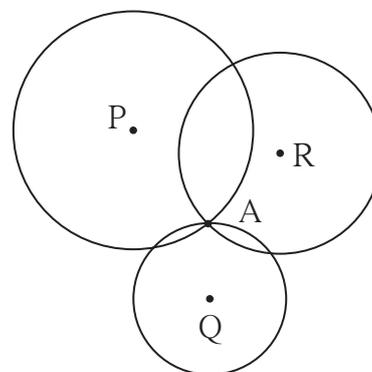


Fig. 3.4



Secant and tangent

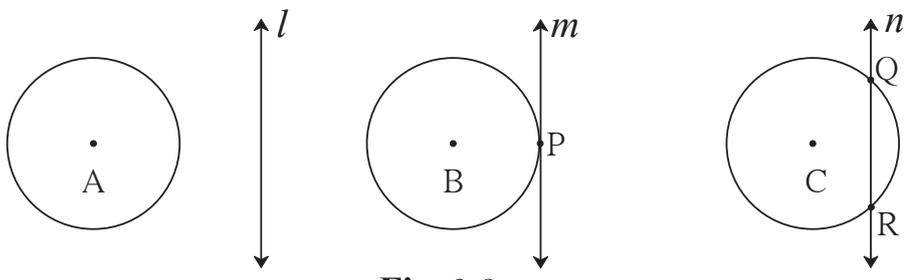


Fig. 3.8

In the figure above, not a single point is common in line l and circle with centre A. Point P is common to both, line m and circle with centre B. Here, line m is called a *tangent* of the circle and point P is called the point of contact.

Two points Q and R are common to both, the line n and the circle with centre C.

Q and R are intersecting points of line n and the circle. Line n is called a *secant* of the circle .

Let us understand an important property of a tangent from the following activity.

Activity :

Draw a sufficiently large circle with centre O. Draw radius OP. Draw a line $AB \perp$ seg OP. It intersects the circle at points A, B. Imagine the line slides towards point P such that all the time it remains parallel to its original position. Obviously, while the line slides, points A and B approach each other along the circle. At the end, they get merged in point P, but the angle between the radius OP and line AB will remain a right angle.

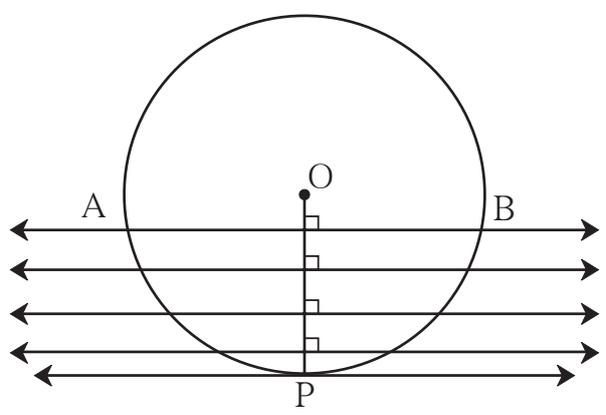


Fig. 3.9

At this stage the line AB becomes a tangent of the circle at P.

So it is clear that, the tangent at any point of a circle is perpendicular to the radius at that point.

This property is known as ‘tangent theorem’.



Which theorems do we use in proving that hypotenuse is the longest side of a right angled triangle?

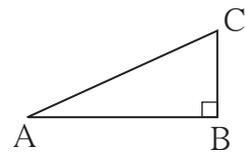


Fig. 3.12



Converse of tangent theorem

Theorem: A line perpendicular to a radius at its point on the circle is a tangent to the circle.

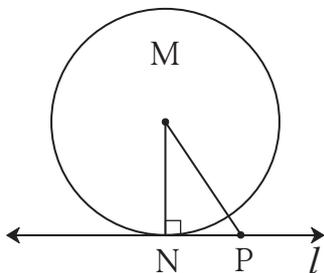


Fig. 3.13

Given : M is the centre of a circle
seg MN is a radius.

Line $l \perp$ seg MN at N.

To prove : Line l is a tangent to the circle.

Proof : Take any point P, other than N, on the line l . Draw seg MP.

Now in ΔMNP , $\angle N$ is a right angle.

\therefore seg MP is the hypotenuse.

\therefore seg MP $>$ seg MN.

As seg MN is radius, point P can't be on the circle.

\therefore no other point, except point N, of line l is on the circle.

\therefore line l intersects the circle in only one point N.

\therefore line l is a tangent to the circle.



In figure 3.14 , B is a point on the circle with centre A. The tangent of the circle passing through B is to be drawn. There are infinite lines passing through the point B. Which of them will be the tangent ? Can the number of tangents through B be more than one ?

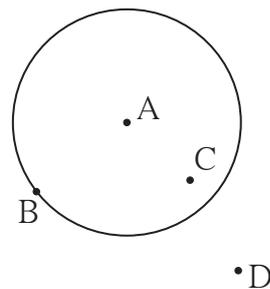


Fig. 3.14

Point C lies in the interior of the circle. Can you draw tangents to the circle through C ?

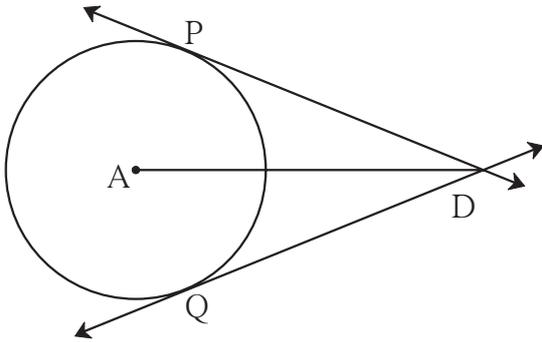


Fig. 3.15

Point D is in the exterior of the circle. Can you draw a tangent to the circle through D? If yes, how many such tangents are possible? From the discussion you must have understood that two tangents can be drawn to a circle from the point outside the circle as shown in the figure.

In the adjoining figure line DP and line DQ, touch the circle at points P and Q. Seg DP and seg DQ are called tangent segments.

Tangent segment theorem

Theorem : Tangent segments drawn from an external point to a circle are congruent.

Observe the adjoining figure. Write ‘given’ and ‘to prove.’

Draw radius AP and radius AQ and complete the following proof of the theorem.

Proof : In $\triangle PAD$ and $\triangle QAD$,
 seg PA \cong _____ radii of the same circle.
 seg AD \cong seg AD _____
 $\angle APD = \angle AQP = 90^\circ$ tangent theorem
 $\therefore \triangle PAD \cong \triangle QAD$ _____
 \therefore seg DP \cong seg DQ _____

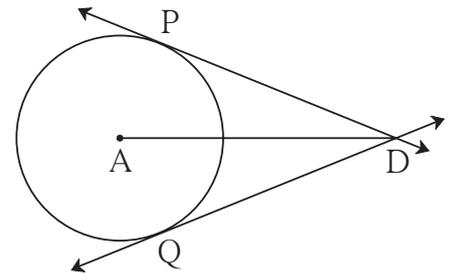


Fig. 3.16

Solved Examples

Ex. (1) In the adjoining figure circle with centre D touches the sides of $\angle ACB$ at A and B. If $\angle ACB = 52^\circ$, find measure of $\angle ADB$.

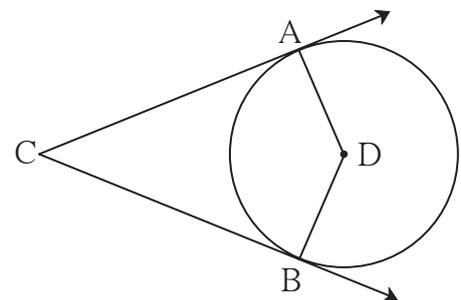


Fig. 3.17

Solution : The sum of all angles of a quadrilateral is 360° .
 $\therefore \angle ACB + \angle CAD + \angle CBD + \angle ADB = 360^\circ$
 $\therefore 52^\circ + 90^\circ + 90^\circ + \angle ADB = 360^\circ$ Tangent theorem
 $\therefore \angle ADB + 232^\circ = 360^\circ$
 $\therefore \angle ADB = 360^\circ - 232^\circ = 128^\circ$

Practice set 3.1

1. In the adjoining figure the radius of a circle with centre C is 6 cm, line AB is a tangent at A. Answer the following questions.

- (1) What is the measure of $\angle CAB$? Why ?
- (2) What is the distance of point C from line AB? Why ?
- (3) $d(A,B) = 6$ cm, find $d(B,C)$.
- (4) What is the measure of $\angle ABC$? Why ?

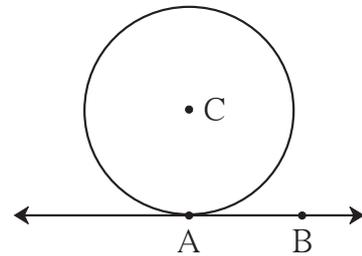


Fig. 3.19

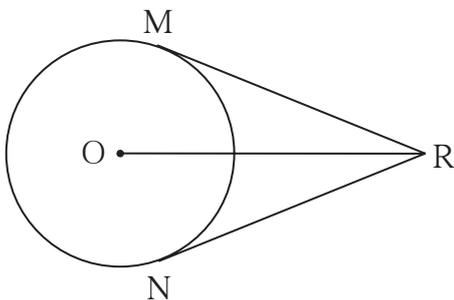


Fig. 3.20

2. In the adjoining figure, O is the centre of the circle. From point R, seg RM and seg RN are tangent segments touching the circle at M and N. If $(OR) = 10$ cm and radius of the circle = 5 cm, then

- (1) What is the length of each tangent segment ?
- (2) What is the measure of $\angle MRO$?
- (3) What is the measure of $\angle MRN$?

3. Seg RM and seg RN are tangent segments of a circle with centre O. Prove that seg OR bisects $\angle MRN$ as well as $\angle MON$.

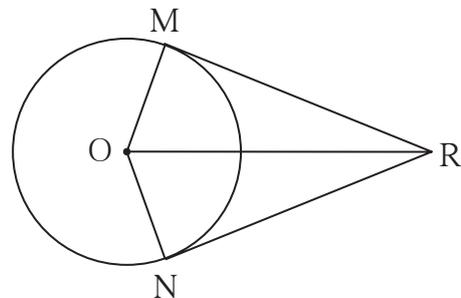


Fig. 3.21

4. What is the distance between two parallel tangents of a circle having radius 4.5 cm ? Justify your answer.



ICT Tools or Links

With the help of Geogebra software, draw a circle and its tangents from a point in its exterior. Check that the tangent segments are congruent.



Let's learn.

Touching circles

Activity I :

Take three collinear points X-Y-Z as shown in figure 3.22. Draw a circle with centre X and radius XY.

Draw another circle with centre Z and radius YZ.

Note that both the circles intersect each other at the single point Y.

Draw a line through point Y and perpendicular to seg XZ.

Note that this line is a common tangent of the two circles.

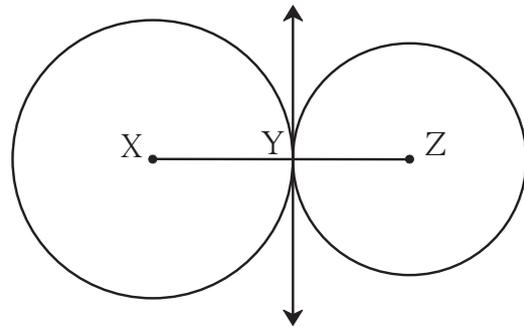


Fig. 3.22

Activity II :

Take points Y-X-Z as shown in the figure 3.23.

Draw a circle with centre Z and radius ZY.

Also draw a circle with centre X and radius XY.

Note that both the circles intersect each other at the point Y.

Draw a line perpendicular to seg YZ through point Y, that is the common tangent for the circles.

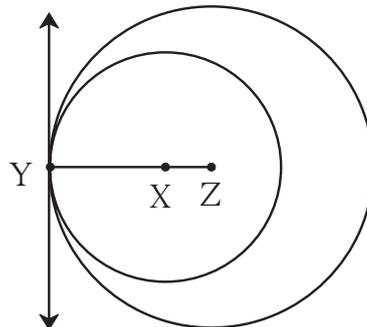


Fig. 3.23

You must have understood, the circles in both the figures are coplaner and intersect at one point only. Such circles are said to be circles touching each other.

Touching circles can be defined as follows.

If two circles in the same plane intersect with a line in the plain in only one point, they are said to be touching circles and the line is their common tangent. The point common to the circles and the line is called their common point of contact.

Given : C is the point of contact of the two circles with centers A, B.

To prove : Point C lies on the line AB.

Proof : Let line l be the common tangent passing through C, of the two touching circles. $\text{line } l \perp \text{seg } AC, \text{line } l \perp \text{seg } BC. \therefore \text{seg } AC \perp \text{line } l \text{ and seg } BC \perp \text{line } l.$
Through C, only one line perpendicular to line l can be drawn.
 \therefore points C, A, B are collinear.



Remember this!

- (1) The point of contact of the touching circles lies on the line joining their centres.
- (2) If the circles touch each other externally, distance between their centres is equal to the sum of their radii.
- (3) The distance between the centres of the circles touching internally is equal to the difference of their radii.

Practice set 3.2

1. Two circles having radii 3.5 cm and 4.8 cm touch each other internally. Find the distance between their centres.
2. Two circles of radii 5.5 cm and 4.2 cm touch each other externally. Find the distance between their centres.
3. If radii of two circles are 4 cm and 2.8 cm. Draw figure of these circles touching each other - (i) externally (ii) internally.

4. In fig 3.27, the circles with centres P and Q touch each other at R. A line passing through R meets the circles at A and B respectively. Prove that -

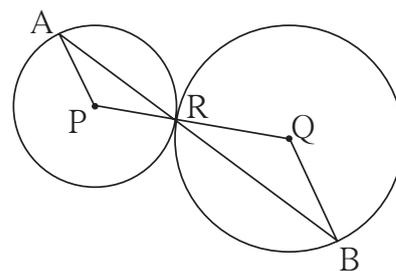


Fig. 3.27

- (1) $\text{seg } AP \parallel \text{seg } BQ,$
- (2) $\Delta APR \sim \Delta RQB,$ and
- (3) Find $\angle RQB$ if $\angle PAR = 35^\circ$

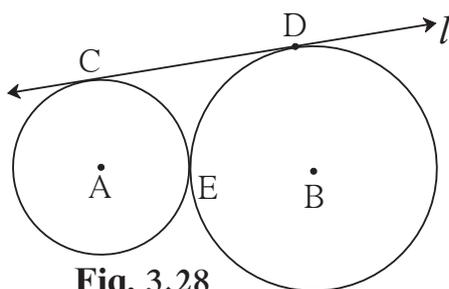


Fig. 3.28

5*. In fig 3.28 the circles with centres A and B touch each other at E. Line l is a common tangent which touches the circles at C and D respectively. Find the length of seg CD if the radii of the circles are 4 cm, 6 cm.

- (1) Measure of a minor arc is equal to the measure of its corresponding central angle. In figure 3.30 measure of central $\angle AOB$ is θ .
 \therefore measure of minor arc APB is also θ .
- (2) Measure of major arc = 360° - measure of corresponding minor arc.
 In figure 3.30 measure of major arc AQB = 360° - measure of minor arc APB
 $= 360^\circ - \theta$
- (3) Measure of a semi circular arc, that is of a semi circle is 180° .
- (4) Measure of a complete circle is 360° .



Let's learn.

Congruence of arcs

When two coplanar figures coincide with each other, they are called congruent figures. We know that two angles of equal measure are congruent.

Similarly, are two arcs of the same measure congruent ?

Find the answer of the question by doing the following activity.

Activity :

Draw two circles with centre C, as shown in the figure. Draw $\angle DCE$, $\angle FCG$ of the same measure and $\angle ICJ$ of different measure.

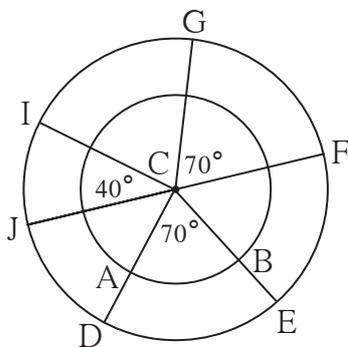


Fig. 3.31

Arms of $\angle DCE$ intersect inner circle at A and B.

Do you notice that the measures of arcs AB and DE are the same ? Do they coincide ? No, definitely not.

Now cut and separate the sectors C-DE; C-FG and C-IJ. Check whether

the arc DE, arc FG and arc IJ coincide with each other.

Did you notice that equality of measures of two arcs is not enough to make the two arcs congruent ? Which additional condition do you think is necessary to make the two arcs congruent ?

From the above activity -

Two arcs are congruent if their measures and radii are equal.

'Arc DE and arc GF are congruent' is written in symbol as $\text{arc DE} \cong \text{arc GF}$.



Ex. (2) In the figure 3.36 a rectangle PQRS is inscribed in a circle with centre T. Prove that, (i) arc PQ \cong arc SR

(ii) arc SPQ \cong arc PQR

Solution : (i) \square PQRS in a rectangle.

\therefore chord PQ \cong chord SR opposite sides of a rectangle
 \therefore arc PQ \cong arc SR arcs corresponding to congruent chords.

(ii) chord PS \cong chord QR Opposite sides of a rectangle

\therefore arc SP \cong arc QR arcs corresponding to congruent chords.

\therefore measures of arcs SP and QR are equal

Now, $m(\text{arc SP}) + m(\text{arc PQ}) = m(\text{arc PQ}) + m(\text{arc QR})$

$\therefore m(\text{arc SPQ}) = m(\text{arc PQR})$

\therefore arc SPQ \cong arc PQR

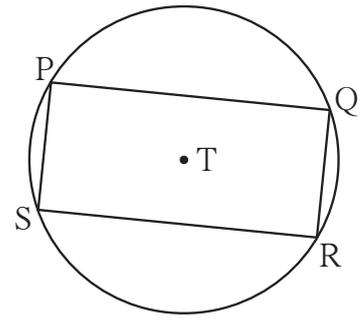


Fig. 3.36



Remember this!

- (1) An angle whose vertex is the centre of a circle is called a central angle.
- (2) Definition of measure of an arc - (i) The measure of a minor arc is the measure of its central angle. (ii) Measure of a major arc = 360° - measure of its corresponding minor arc. (iii) measure of a semicircle is 180° .
- (3) When two arcs are of the same radius and same measure, they are congruent.
- (4) When only one point C is common to arc ABC, and arc CDE of the same circle, $m(\text{arc ABC}) + m(\text{arc CDE}) = m(\text{arc ACE})$
- (5) Chords of the same or congruent circles are equal if the related arcs are congruent.
- (6) Arcs of the same or congruent circles are equal if the related chords are congruent.

Practice set 3.3

1. In figure 3.37, points G, D, E, F are concyclic points of a circle with centre C.

$\angle ECF = 70^\circ$, $m(\text{arc DGF}) = 200^\circ$

find $m(\text{arc DE})$ and $m(\text{arc DEF})$.

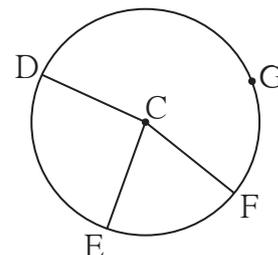


Fig. 3.37

Intercepted arc

Observe all figures (i) to (vi) in the following figure 3.43.

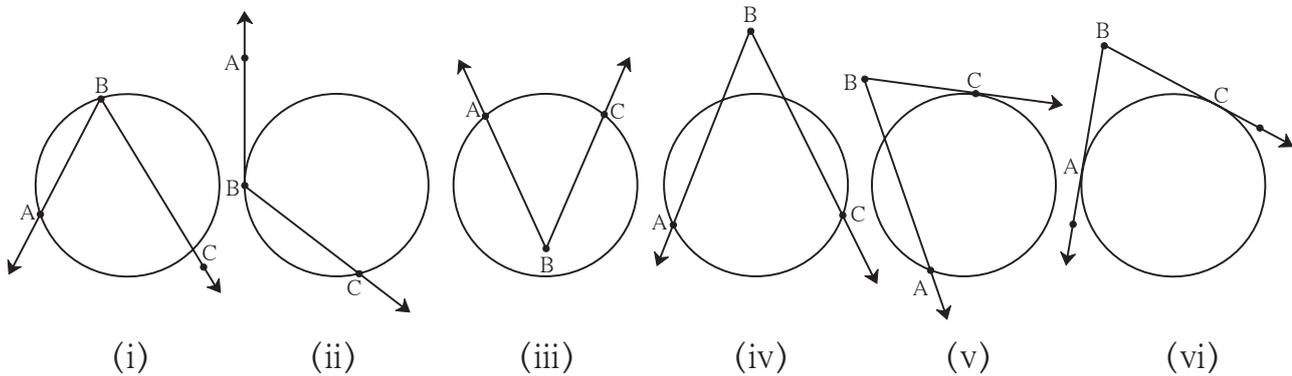


Fig. 3.43

In each figure, the arc of a circle that lies in the interior of the $\angle ABC$ is an arc intercepted by the $\angle ABC$. The points of intersection of the circle and the angle are end points of that intercepted arc. Each side of the angle has to contain an end point of the arc.

In figures 3.43 (i), (ii) and (iii) only one arc is intercepted by that angle; and in (iv), (v) and (vi), two arcs are intercepted by the angle.

Also note that, only one side of the angle touches the circle in (ii) and (v), but in (vi) both sides of the angle touch the circle.

In figure 3.44, the arc is not intercepted arc, as arm BC does not contain any end point of the arc.

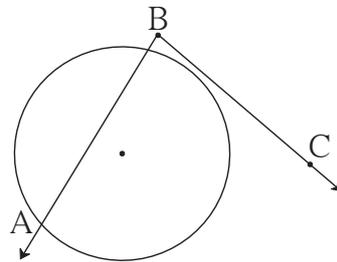


Fig. 3.44

Inscribed angle theorem

The measure of an inscribed angle is half of the measure of the arc intercepted by it.

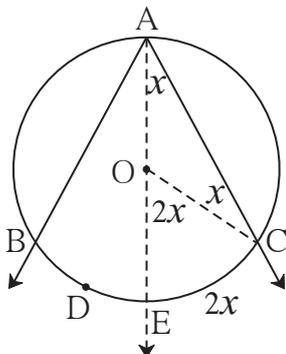


Fig. 3.45

Given : In a circle with centre O, $\angle BAC$ is inscribed in arc BAC. Arc BDC is intercepted by the angle.

To prove: $\angle BAC = \frac{1}{2} m(\text{arc BDC})$

Construction : Draw ray AO. It intersects the circle at E. Draw radius OC.

Corollaries of inscribed angle theorem

1. Angles inscribed in the same arc are congruent.

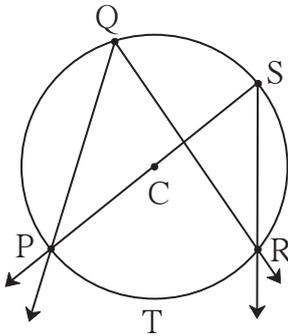


Fig. 3.47

Write ‘given’ and ‘to prove’ with the help of the figure 3.47.

Think of the answers of the following questions and write the proof.

- (1) Which arc is intercepted by $\angle PQR$?
- (2) Which arc is intercepted by $\angle PSR$?
- (3) What is the relation between an inscribed angle and the arc intercepted by it ?

2. Angle inscribed in a semicircle is a right angle.

With the help of figure 3.48 write ‘given’, ‘to prove’ and ‘the proof’.

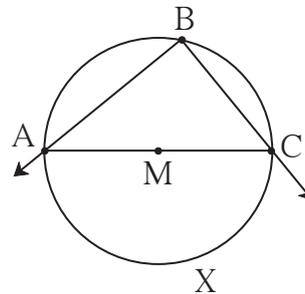


Fig. 3.48

Cyclic quadrilateral

If all vertices of a quadrilateral lie on the same circle then it is called a cyclic quadrilateral.

Theorem of cyclic quadrilateral

Theorem: Opposite angles of a cyclic quadrilateral are supplementary.

Fill in the blanks and complete the following proof.

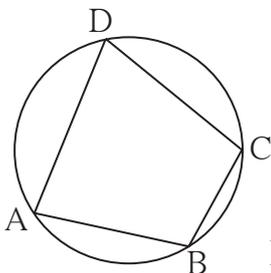


Fig. 3.49

Given : is cyclic.

To prove: $\angle B + \angle D =$
 + $\angle C = 180^\circ$

Proof : Arc ABC is intercepted by the inscribed angle $\angle ADC$.

$\therefore \angle ADC = \frac{1}{2}$ (I)

Similarly, is an inscribed angle. It intercepts arc ADC.

Theorem : If two points on a given line subtend equal angles at two distinct points which lie on the same side of the line, then the four points are concyclic.

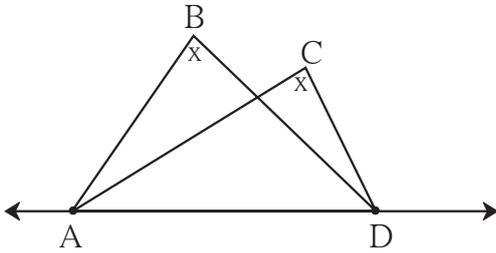


Fig. 3.50

Given : Points B and C lie on the same side of the line AD. $\angle ABD \cong \angle ACD$

To prove: Points A, B, C, D are concyclic. (That is, $\square ABCD$ is cyclic.)

This theorem can be proved by 'indirect method'.



Let's think.

The above theorem is converse of a certain theorem. State it.

Solved Examples

Ex. (1) In figure 3.51, chord $LM \cong$ chord LN

$\angle L = 35^\circ$ find

(i) $m(\text{arc } MN)$

(ii) $m(\text{arc } LN)$

Solution : (i) $\angle L = \frac{1}{2} m(\text{arc } MN)$ inscribed angle theorem.

$$\therefore 35 = \frac{1}{2} m(\text{arc } MN)$$

$$\therefore 2 \times 35 = m(\text{arc } MN) = 70^\circ$$

(ii) $m(\text{arc } MLN) = 360^\circ - m(\text{arc } MN)$ definition of measure of arc
 $= 360^\circ - 70^\circ = 290^\circ$

Now, chord $LM \cong$ chord LN

\therefore arc $LM \cong$ arc LN

but $m(\text{arc } LM) + m(\text{arc } LN) = m(\text{arc } MLN) = 290^\circ$ arc addition property

$$m(\text{arc } LM) = m(\text{arc } LN) = \frac{290^\circ}{2} = 145^\circ$$

or, (ii) chord $LM \cong$ chord LN

$\therefore \angle M = \angle N$ isosceles triangle theorem.

$$\therefore 2 \angle M = 180^\circ - 35^\circ = 145^\circ$$

$$\therefore \angle M = \frac{145^\circ}{2}$$

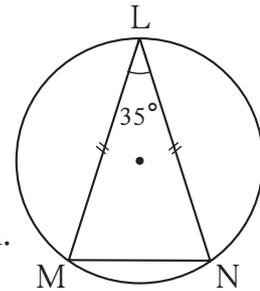


Fig. 3.51

Ex. (3) Prove that, if two lines containing chords of a circle intersect each other outside the circle, then the measure of angle between them is half the difference in measures of the arcs intercepted by the angle.

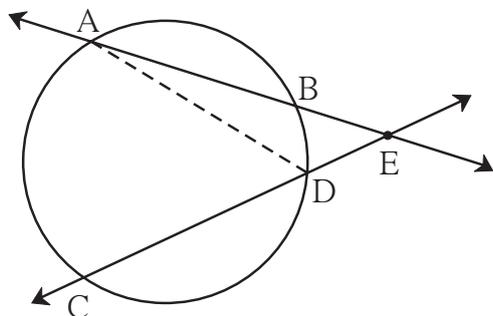


Fig. 3.53

Given : Chords AB and CD intersect at E in the exterior of the circle.

To prove: $\angle AEC = \frac{1}{2} [m(\text{arc } AC) - m(\text{arc } BD)]$

Construction: Draw seg AD.

Consider angles of $\triangle AED$ and its exterior angle and write the proof.



Remember this!

- (1) The measure of an inscribed angle is half the measure of the arc intercepted by it.
- (2) Angles inscribed in the same arc are congruent.
- (3) Angle inscribed in a semicircle is a right angle.
- (4) If all vertices of a quadrilateral lie on the same circle then the quadrilateral is called a cyclic quadrilateral.
- (5) Opposite angles of a cyclic quadrilateral are supplementary.
- (6) An exterior angle of a cyclic quadrilateral is congruent to the angle opposite to its adjacent interior angle.
- (7) If a pair of opposite angles of a quadrilateral is supplementary, then the quadrilateral is cyclic.
- (8) If two points on a given line subtend equal angles at two different points which lie on the same side of the line, then those four points are concyclic.

(9) In figure 3.54,

(i) $\angle AEC = \frac{1}{2} [m(\text{arc } AC) + m(\text{arc } DB)]$

(ii) $\angle CEB = \frac{1}{2} [m(\text{arc } AD) + m(\text{arc } CB)]$

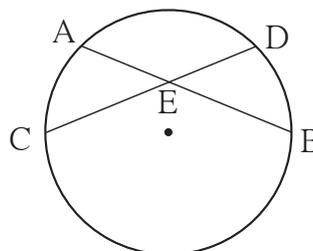


Fig. 3.54

- (10) In figure 3.55,

$$\angle BED = \frac{1}{2} [m(\text{arc } BD) - m(\text{arc } AC)]$$

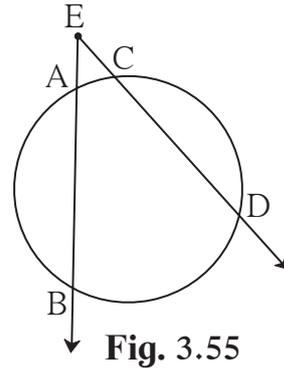


Fig. 3.55

Practice set 3.4

1. In figure 3.56, in a circle with centre O, length of chord AB is equal to the radius of the circle. Find measure of each of the following.

- (1) $\angle AOB$ (2) $\angle ACB$
 (3) arc AB (4) arc ACB.

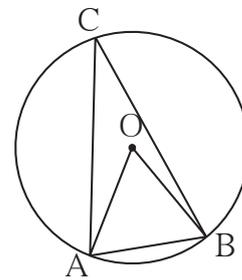


Fig. 3.56

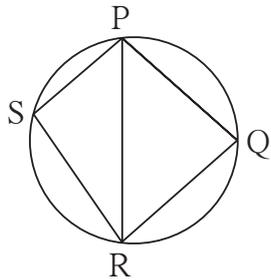


Fig. 3.57

2. In figure 3.57, $\square PQRS$ is cyclic. side $PQ \cong$ side RQ . $\angle PSR = 110^\circ$, Find-
- (1) measure of $\angle PQR$
 (2) $m(\text{arc } PQR)$
 (3) $m(\text{arc } QR)$
 (4) measure of $\angle PRQ$

3. $\square MRPN$ is cyclic, $\angle R = (5x - 13)^\circ$, $\angle N = (4x + 4)^\circ$. Find measures of $\angle R$ and $\angle N$.

4. In figure 3.58, seg RS is a diameter of the circle with centre O. Point T lies in the exterior of the circle. Prove that $\angle RTS$ is an acute angle.

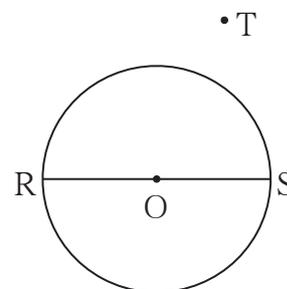


Fig. 3.58

5. Prove that, any rectangle is a cyclic quadrilateral.

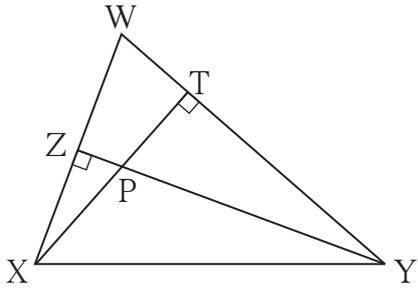


Fig. 3.59

6. In figure 3.59, altitudes YZ and XT of ΔWXY intersect at P. Prove that,
 (1) $\square WZPT$ is cyclic.
 (2) Points X, Z, T, Y are concyclic.

7. In figure 3.60, $m(\text{arc NS}) = 125^\circ$,
 $m(\text{arc EF}) = 37^\circ$, find the measure $\angle NMS$.

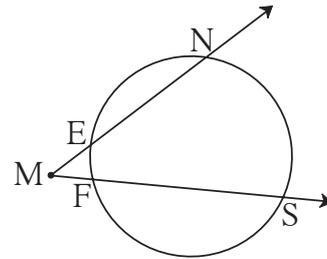


Fig. 3.60

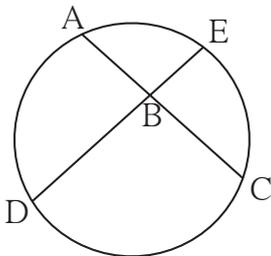


Fig. 3.61

8. In figure 3.61, chords AC and DE intersect at B. If $\angle ABE = 108^\circ$,
 $m(\text{arc AE}) = 95^\circ$, find $m(\text{arc DC})$.

 **Let's learn.**

Activity :

Draw a circle as shown in figure 3.62. Draw a chord AC. Take any point B on the circle. Draw inscribed $\angle ABC$, measure it and note the measure.

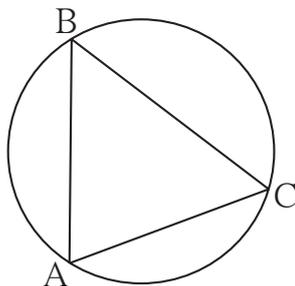


Fig. 3.62

Now as shown in figure 3.63, draw a tangent CD of the same circle, measure angle $\angle ACD$ and note the measure.

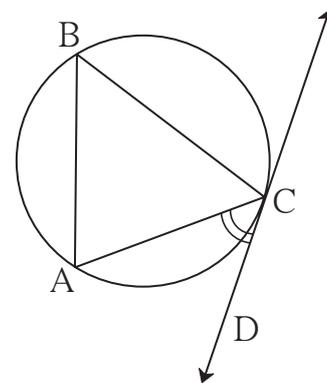


Fig. 3.63

You will find that $\angle ACD = \angle ABC$.

You know that $\angle ABC = \frac{1}{2} m(\text{arc AC})$

From this we get $\angle ACD = \frac{1}{2} m(\text{arc AC})$.

Now we will prove this property.

Theorem of angle between tangent and secant

If an angle has its vertex on the circle, its one side touches the circle and the other intersects the circle in one more point, then the measure of the angle is half the measure of its intercepted arc.

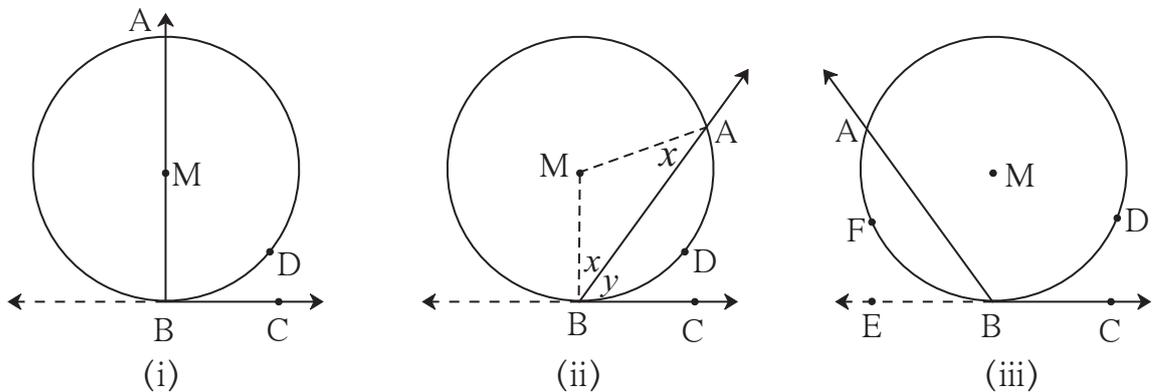


Fig. 3.64

Given : Let $\angle ABC$ be an angle, where vertex B lies on a circle with centre M.

Its side BC touches the circle at B and side BA intersects the circle at A. Arc ADB is intercepted by $\angle ABC$.

To prove: $\angle ABC = \frac{1}{2} m(\text{arc ADB})$

Proof : Consider three cases.

(1) In figure 3.64 (i), the centre M lies on the arm BA of $\angle ABC$,

$\angle ABC = \angle MBC = 90^\circ$ tangent theorem (I)

arc ADB is a semicircle.

$\therefore m(\text{arc ADB}) = 180^\circ$ definition of measure of arc (II)

From (I) and (II)

$$\angle ABC = \frac{1}{2} m(\text{arc ADB})$$

(2) In figure 3.64 (ii) centre M lies in the exterior of $\angle ABC$,

Draw radii MA and MB.

Now, $\angle MBA = \angle MAB$ isosceles triangle theorem

$\angle MBC = 90^\circ$ tangent theorem..... (I)

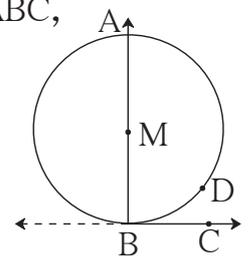


Fig. 3.64(i)

let $\angle MBA = \angle MAB = x$ and $\angle ABC = y$.

$$\angle AMB = 180 - (x + x) = 180 - 2x$$

$$\angle MBC = \angle MBA + \angle ABC = x + y$$

$$\therefore x + y = 90^\circ \quad \therefore 2x + 2y = 180^\circ$$

$$\text{In } \triangle AMB, 2x + \angle AMB = 180^\circ$$

$$\therefore 2x + 2y = 2x + \angle AMB$$

$$\therefore 2y = \angle AMB$$

$$\therefore y = \angle ABC = \frac{1}{2} \angle AMB = \frac{1}{2} m(\text{arc ADB})$$

(3) With the help of fig 3.64 (iii),

Fill in the blanks and write proof.

Ray is the opposite ray of ray BC.

Now, $\angle ABE = \frac{1}{2} m(\quad)$ proved in (ii).

$$\begin{aligned} \therefore 180 - \text{input} &= \frac{1}{2} m(\text{arc AFB}) \dots \text{linear pair} \\ &= \frac{1}{2} [360 - m(\text{input})] \end{aligned}$$

$$\therefore 180 - \angle ABC = 180 - \frac{1}{2} m(\text{arc ADB})$$

$$\therefore -\angle ABC = -\frac{1}{2} m(\text{input})$$

$$\therefore \angle ABC = \frac{1}{2} m(\text{arc ADB})$$

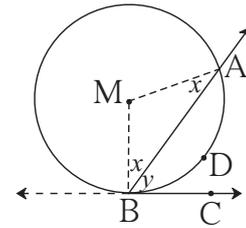


Fig. 3.64(ii)

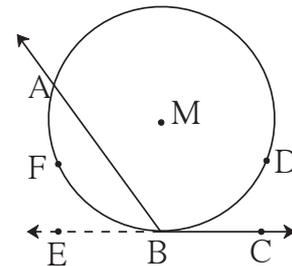


Fig. 3.64(iii)

Alternative statement of the above theorem.

In the figure 3.65, line AB is a secant and line BC is a tangent. The arc ADB is intercepted by $\angle ABC$. Chord AB divides the circle in two parts. These are opposite

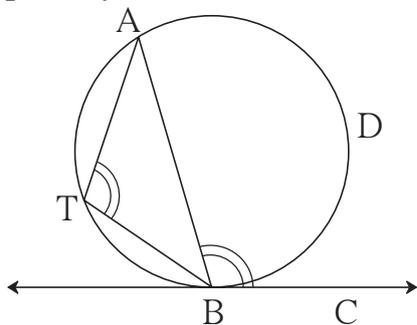


Fig. 3.65

arcs of each other.

Now take any point T on the arc opposite to arc ADB .

From the above theorem,

$$\angle ABC = \frac{1}{2} m(\text{arc ADB}) = \angle ATB.$$

\therefore the angle between a tangent of a circle and a chord drawn from the point of contact is congruent to the angle inscribed in the arc opposite to the arc intercepted by that angle.

Converse of theorem of the angle between tangent and secant

A line is drawn from one end point of a chord of a circle and if the angle between the chord and the line is half the measure of the arc intercepted by that angle then that line is a tangent to the circle.

In figure 3.66,

$$\text{If } \angle PQR = \frac{1}{2} m(\text{arc PSQ}),$$

$$[\text{or } \angle PQT = \frac{1}{2} m(\text{arc PUQ})]$$

then line TR is a tangent to the circle.

This property is used in constructing a tangent to the given circle.

An indirect proof of this converse can be given.

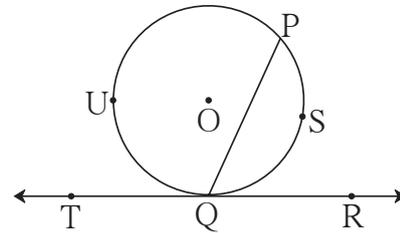


Fig. 3.66

Theorem of internal division of chords

Suppose two chords of a circle intersect each other in the interior of the circle, then the product of the lengths of the two segments of one chord is equal to the product of the lengths of the two segments of the other chord.

Given : Chords AB and CD of a circle with centre P intersect at point E.

To prove: $AE \times EB = CE \times ED$

Construction : Draw seg AC and seg DB.

Proof : In $\triangle CAE$ and $\triangle BDE$,

$$\angle AEC \cong \angle DEB \quad \dots \text{ opposite angles}$$

$$\angle CAE \cong \angle BDE \quad \dots \text{ angles inscribed in the same arc}$$

$$\therefore \triangle CAE \sim \triangle BDE \quad \dots \text{ AA test}$$

$$\therefore \frac{AE}{DE} = \frac{CE}{BE} \quad \dots \text{ corresponding sides of similar triangles}$$

$$\therefore AE \times EB = CE \times ED$$

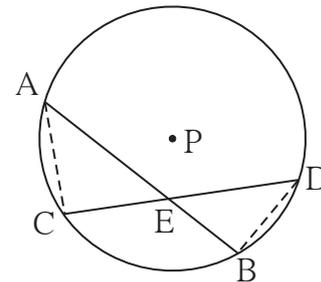


Fig. 3.67



Let's think.

We proved the theorem by drawing seg AC and seg DB in figure 3.67, Can the theorem be proved by drawing seg AD and seg CB, instead of seg AC and seg DB?

For more information

In figure 3.67 point E divides the chord AB into segments AE and EB. $AE \times EB$ is the area of a rectangle having sides AE and EB. Similarly E divides CD into segments CE and ED. $CE \times ED$ is the area of a rectangle of sides CE and ED. We have proved that $AE \times EB = CE \times ED$.

So the above theorem can be stated as, ‘If two chords of a circle intersect in the interior of a circle then the area of the rectangle formed by the segments of one chord is equal to the area of similar rectangle formed by the other chord.’

Theorem of external division of chords

If secants containing chords AB and CD of a circle intersect outside the circle in point E, then $AE \times EB = CE \times ED$.

Write ‘given’ and ‘to prove’ with the help of the statement of the theorem and figure 3.68.

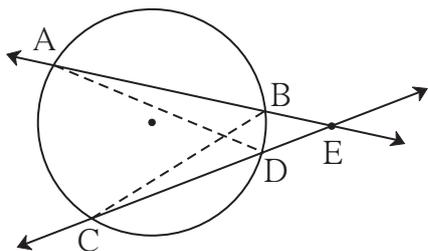


Fig. 3.68

Construction : Draw seg AD and seg BC.

Fill in the blanks and complete the proof.

Proof : In ΔADE and ΔCBE ,

$\angle AED \cong$ common angle

$\angle DAE \cong \angle BCE$ ()

$\therefore \Delta ADE \sim$ ()

$\therefore \frac{(AE)}{\text{}} = \frac{\text{}}{\text{}}$ corresponding sides of similar triangles

$\therefore \text{} = CE \times ED$

Tangent secant segments theorem

Point E is in the exterior of a circle. A secant through E intersects the circle at points A and B, and a tangent through E touches the circle at point T, then $EA \times EB = ET^2$.

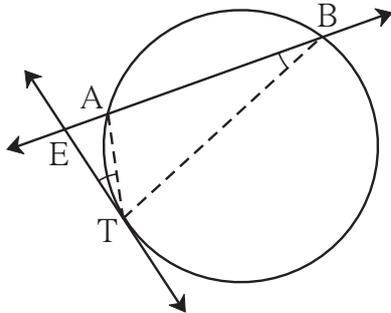


Fig. 3.69

Write 'given' and 'to prove' with reference to the statement of the theorem.

Construction : Draw seg TA and seg TB.

Proof : In $\triangle EAT$ and $\triangle ETB$,

$\angle AET \cong \angle TEB$ common angle

$\angle ETA \cong \angle EBT$... tangent secant theorem

$\therefore \triangle EAT \sim \triangle ETB$ AA similarity

$\therefore \frac{ET}{EB} = \frac{EA}{ET}$ corresponding sides

$\therefore EA \times EB = ET^2$



Remember this!

- (1) In figure 3.70,
 $AE \times EB = CE \times ED$
 This property is known as theorem of chords intersecting inside the circle.

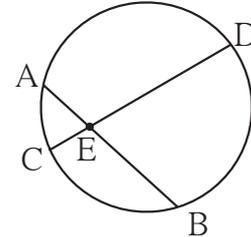


Fig. 3.70

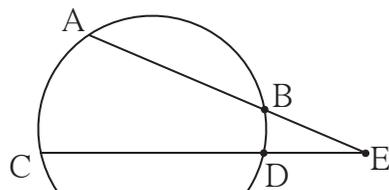


Fig. 3.71

- (2) In figure 3.71,
 $AE \times EB = CE \times ED$
 This property is known as theorem of chords intersecting outside the circle.

- (3) In figure 3.72,
 $EA \times EB = ET^2$
 This property is known as tangent secant segments theorem.

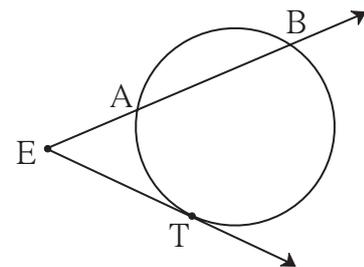


Fig. 3.72

Solved Examples

Ex. (1) In figure 3.73, seg PS is a tangent segment.
 Line PR is a secant.
 If $PQ = 3.6$,
 $QR = 6.4$, find PS.

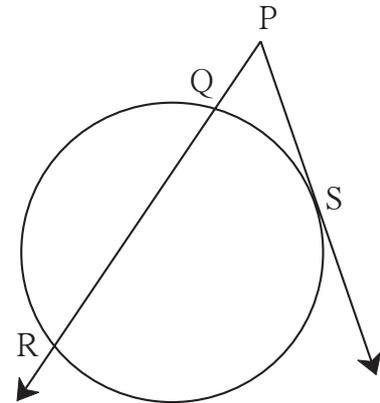


Fig. 3.73

Solution : $PS^2 = PQ \times PR$ tangent secant segments theorem

$$\begin{aligned} &= PQ \times (PQ + QR) \\ &= 3.6 \times [3.6 + 6.4] \\ &= 3.6 \times 10 \\ &= 36 \\ \therefore PS &= 6 \end{aligned}$$

Ex. (2)

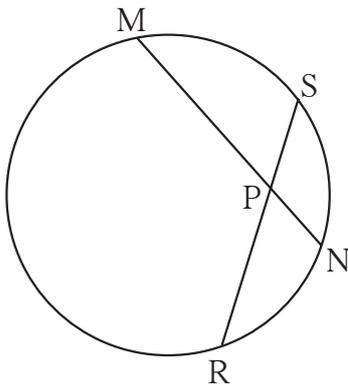


Fig. 3.74

In figure 3.74, chord MN and chord RS intersect each other at point P.
 If $PR = 6$, $PS = 4$, $MN = 11$
 find PN.

Solution : By theorem on intersecting chords,

$$PN \times PM = PR \times PS \dots (I)$$

$$\text{let } PN = x. \therefore PM = 11 - x$$

substituting the values in (I),

$$x(11 - x) = 6 \times 4$$

$$\therefore 11x - x^2 - 24 = 0$$

$$\therefore x^2 - 11x + 24 = 0$$

$$\therefore (x - 3)(x - 8) = 0$$

$$\therefore x - 3 = 0 \text{ or } x - 8 = 0$$

$$\therefore x = 3 \text{ or } x = 8$$

$$\therefore PN = 3 \text{ or } PN = 8$$

Ex. (3) In figure 3.75, two circles intersect each other in points X and Y. Tangents drawn from a point M on line XY touch the circles at P and Q.

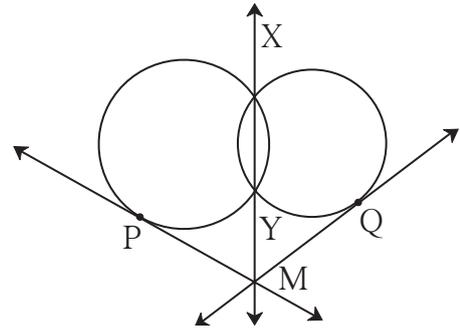


Fig. 3.75

Prove that, $\text{seg } PM \cong \text{seg } QM$.

Solution : Fill in the blanks and write proof.

Line MX is a common of the two circles.

$$\therefore PM^2 = MY \times MX \dots\dots (I)$$

Similarly = \times , tangent secant segment theorem(II)

$$\therefore \text{From (I) and (II) } \dots\dots = QM^2$$

$$\therefore PM = QM$$

$$\text{seg } PM \cong \text{seg } QM$$

Ex. (4)

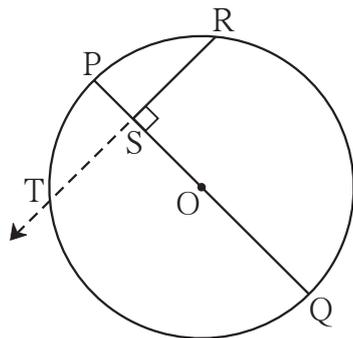


Fig. 3.76

In figure 3.76, seg PQ is a diameter of a circle with centre O. R is any point on the circle.

seg RS \perp seg PQ.

Prove that, SR is the geometric mean of PS and SQ.

$$[\text{That is, } SR^2 = PS \times SQ]$$

Solution : Write the proof with the help of the following steps.

- (1) Draw ray RS. It intersects the circle at T.
- (2) Show that RS = TS.
- (3) Write a result using theorem of intersection of chords inside the circle.
- (4) Using RS = TS complete the proof.



Let's think.

- (1) In figure 3.76, if seg PR and seg RQ are drawn, what is the nature of ΔPRQ ?
- (2) Have you previously proved the property proved in example (4) ?

Practice set 3.5

1. In figure 3.77, ray PQ touches the circle at point Q. $PQ = 12$, $PR = 8$, find PS and RS.

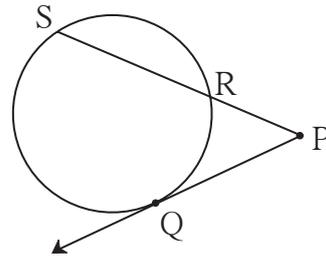


Fig. 3.77

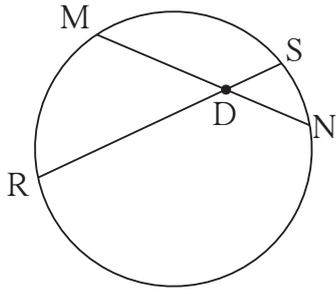


Fig. 3.78

2. In figure 3.78, chord MN and chord RS intersect at point D.
 (1) If $RD = 15$, $DS = 4$, $MD = 8$ find DN
 (2) If $RS = 18$, $MD = 9$, $DN = 8$ find DS

3. In figure 3.79, O is the centre of the circle and B is a point of contact. $\text{seg } OE \perp \text{seg } AD$, $AB = 12$, $AC = 8$, find
 (1) AD (2) DC (3) DE.

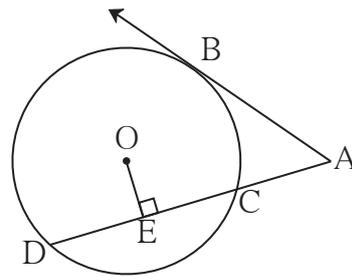


Fig. 3.79

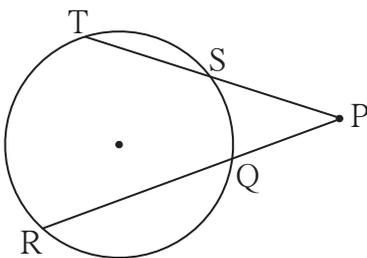


Fig. 3.80

4. In figure 3.80, if $PQ = 6$, $QR = 10$, $PS = 8$ find TS.

5. In figure 3.81, seg EF is a diameter and seg DF is a tangent segment. The radius of the circle is r . Prove that, $DE \times GE = 4r^2$

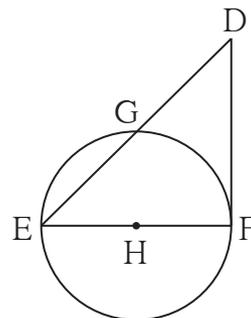


Fig. 3.81

Problem set 3

1. Four alternative answers for each of the following questions are given. Choose the correct alternative.
- (1) Two circles of radii 5.5 cm and 3.3 cm respectively touch each other. What is the distance between their centers ?
 (A) 4.4 cm (B) 8.8 cm (C) 2.2 cm (D) 8.8 or 2.2 cm
- (2) Two circles intersect each other such that each circle passes through the centre of the other. If the distance between their centres is 12, what is the radius of each circle ?
 (A) 6 cm (B) 12 cm (C) 24 cm (D) can't say
- (3) A circle touches all sides of a parallelogram. So the parallelogram must be a,
 (A) rectangle (B) rhombus (C) square (D) trapezium
- (4) Length of a tangent segment drawn from a point which is at a distance 12.5 cm from the centre of a circle is 12 cm, find the diameter of the circle.
 (A) 25 cm (B) 24 cm (C) 7 cm (D) 14 cm
- (5) If two circles are touching externally, how many common tangents of them can be drawn?
 (A) One (B) Two (C) Three (D) Four
- (6) $\angle ACB$ is inscribed in arc ACB of a circle with centre O . If $\angle ACB = 65^\circ$, find $m(\text{arc } ACB)$.
 (A) 65° (B) 130° (C) 295° (D) 230°
- (7) Chords AB and CD of a circle intersect inside the circle at point E . If $AE = 5.6$, $EB = 10$, $CE = 8$, find ED .
 (A) 7 (B) 8 (C) 11.2 (D) 9
- (8) In a cyclic $\square ABCD$, twice the measure of $\angle A$ is thrice the measure of $\angle C$. Find the measure of $\angle C$?
 (A) 36 (B) 72 (C) 90 (D) 108
- 9)* Points A, B, C are on a circle, such that $m(\text{arc } AB) = m(\text{arc } BC) = 120^\circ$. No point, except point B , is common to the arcs. Which is the type of ΔABC ?
 (A) Equilateral triangle (B) Scalene triangle
 (C) Right angled triangle (D) Isosceles triangle

\therefore from (I), (II), (III),
 $\angle XAZ = \dots\dots\dots$
 \therefore radius $XA \parallel$ radius $YB \dots\dots\dots$ ($\dots\dots\dots$)

8. In figure 3.88, circles with centres X and Y touch internally at point Z. Seg BZ is a chord of bigger circle and it intersects smaller circle at point A.

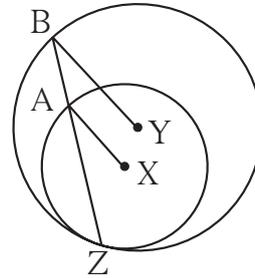


Fig. 3.88

Prove that, seg $AX \parallel$ seg BY .

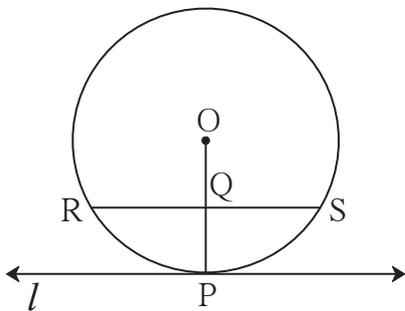


Fig. 3.89

9. In figure 3.89, line l touches the circle with centre O at point P. Q is the mid point of radius OP. RS is a chord through Q such that chords $RS \parallel$ line l . If $RS = 12$ find the radius of the circle.

10*. In figure 3.90, seg AB is a diameter of a circle with centre C. Line PQ is a tangent, which touches the circle at point T.

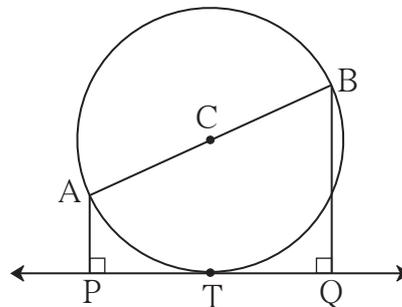


Fig. 3.90

seg $AP \perp$ line PQ and seg $BQ \perp$ line PQ .
 Prove that, seg $CP \cong$ seg CQ .

11*. Draw circles with centres A, B and C each of radius 3 cm, such that each circle touches the other two circles.

12*. Prove that any three points on a circle cannot be collinear.

