5 Co-ordinate Geometry

Let's study.

- Distance formula
- Section formula
- Slope of a line

Let's recall.

We know how to find the distance between any two points on a number line.

If co-ordinates of points P,Q and R are −1,−5 and 4 respectively then find the length of seg PQ, seg QR.

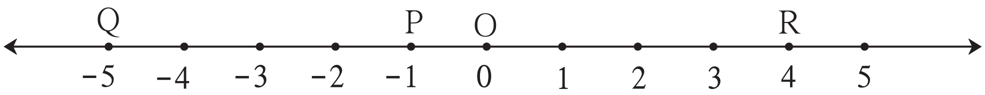


Fig. 5.1

If x_1 and x_2 are the co-ordinates of points A and B and $x_2 > x_1$ then length of seg AB = d(A,B) = $x_2 - x_1$

As shown in the figure, co-ordinates of points P,Q and R are −1,−5 and 4 respectively.

$\therefore$ d(P, Q) = (−1)−(−5) = −1 + 5 = 4

and d(Q, R) = 4 − (−5) = 4 + 5 = 9

Using the same concept we can find the distance between two points on the same axis in XY−plane.

Let's learn.

(1) To find distance between any two points on an axis .

Two points on an axis are like two points on the number line. Note that points on the X−axis have co-ordinates such as (2, 0), $(\frac{-5}{2}, 0)$, (8, 0). Similarly points on the Y-axis have co-ordinates such as (0, 1), $(0, \frac{17}{2})$, (0, −3). Part of the X-axis which shows negative co-ordinates is OX′ and part of the Y-axis which shows negative co-ordinates is OY′.

i) To find distance between two points on X-axis.

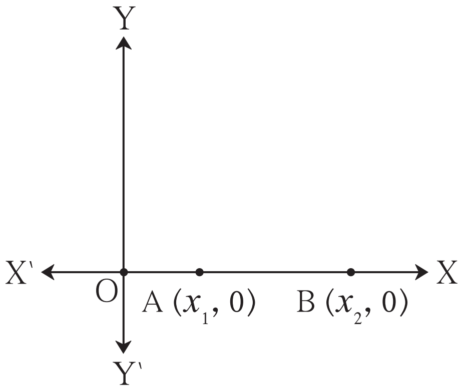


Fig. 5.2

In the above figure, points $A(x_1, 0)$ and $B(x_2, 0)$ are on X- axis such that, $x_2 > x_1$

$\therefore$ $d(A, B) = x_2 - x_1$

ii) To find distance between two points on Y-axis.

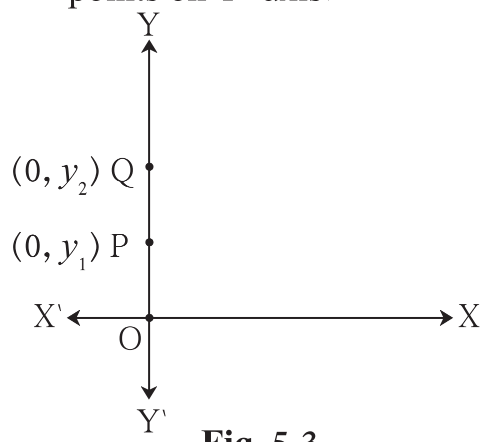


Fig. 5.3

In the above figure, points $P(0, y_1)$ and $Q(0, y_2)$ are on Y- axis such that, $y_2 > y_1$

$\therefore$ $d(P,Q) = y_2 - y_1$

(2) To find the distance between two points if the segment joining these points is parallel to any axis in the XY plane.

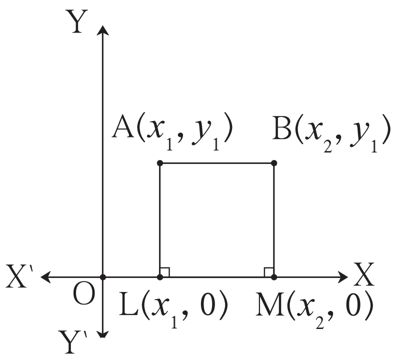


Fig. 5.4

i) In the figure, seg AB is parallel to X- axis.

$\therefore$ y co-ordinates of points A and B are equal

Draw seg AL and seg BM perpendicular to X-axis

$\therefore$ □ ABML is a rectangle.

$\therefore$ AB = LM

But, $LM = x_2 - x_1$

$\therefore$ $d(A,B) = x_2 - x_1$

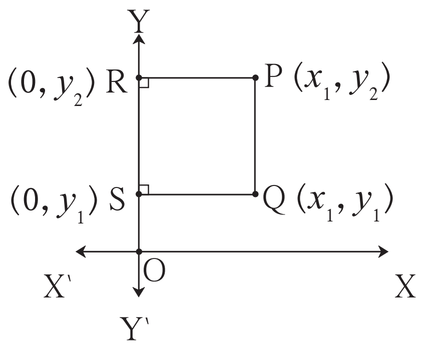


Fig. 5.5

ii) In the figure seg PQ is parallel to Y- axis.

$\therefore$ x co-ordinates of points P and Q are equal

Draw seg PR and seg QS perpendicular to Y-axis.

$\therefore$ □ PQSR is a rectangle

$\therefore$ PQ = RS

But, $RS = y_2 - y_1$

$\therefore$ $d(P,Q) = y_2 - y_1$

Activity:

In the figure, seg AB || Y-axis and seg CB || X-axis. Co-ordinates of points A and C are given.

To find AC, fill in the boxes given below.

Δ ABC is a right angled triangle.

According to Pythagoras theorem,

$(AB)^2 + (BC)^2 = \square$

We will find co-ordinates of point B to find the lengths AB and BC,

CB || X- axis $\therefore$ y co-ordinate of B = $\square$

BA || Y- axis $\therefore$ x co-ordinate of B = $\square$

AB = $\boxed{3} - \square = \square$ BC = $\square - \square = \boxed{4}$

$\therefore$ AC^2 = $\square + \square = \square$ $\therefore$ AC = $\boxed{\sqrt{17}}$

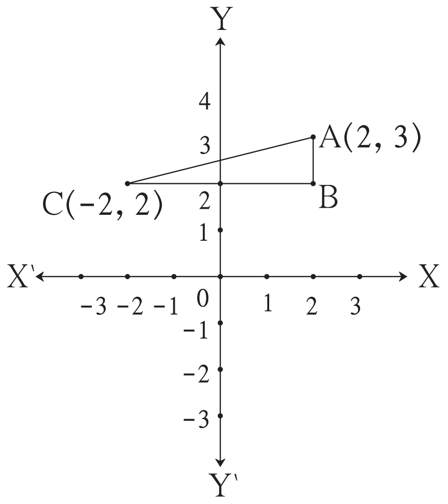


Fig. 5.6

Let's learn.

Distance formula

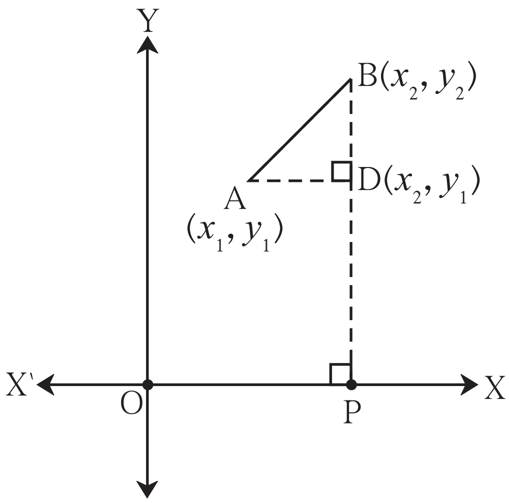


Fig. 5.7

In the figure 5.7, A(x_1, y_1) and B(x_2, y_2) are any two points in the XY plane.

From point B draw perpendicular BP on X-axis. Similarly from point A draw perpendicular AD on seg BP.

seg BP is parallel to Y-axis.

$\therefore$ the x co-ordinate of point D is x_2.

seg AD is parallel to X-axis.

$\therefore$ the y co-ordinate of point D is y_1.

$\therefore$ AD = d(A, D) = $x_2 - x_1$; BD = d(B, D) = $y_2 - y_1$

In right angled triangle Δ ABD, $AB^2 = AD^2 + BD^2$

$$= (x_2 - x_1)^2 + (y_2 - y_1)^2$$

$$\therefore AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

This is known as distance formula.

Note that, $\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}=\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

In the previous activity, we found the lengths of seg AB and seg AC and then used Pythagoras theorem to find the length of seg AC.

Now we will use distance formula to find AC.

A(2, 3) and C(−2, 2) is given

Let $A(x_1, y_1)$ and $C(x_2, y_2)$.

$x_1 = 2, y_1 = 3, x_2 = -2, y_2 = 2$

$AC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{(-2-2)^2+(2-3)^2}$

$= \sqrt{(-4)^2+(-1)^2}$

$= \sqrt{16+1}$

$= \sqrt{17}$

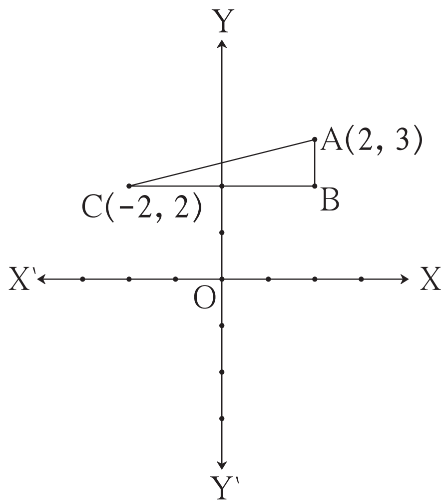


Fig. 5.8

seg AB || Y−axis and seg BC || X−axis.

∴ co-ordinates of point B are (2, 2).

$\therefore AB = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(2-2)^2+(2-3)^2} = \sqrt{0+1} = 1$

$BC = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2} = \sqrt{(-2-2)^2+(2-2)^2} = \sqrt{(-4)^2+0} = 4$

In the Figure 5.1, distance between points P and Q is found as (−1) − (−5) = 4. In XY- plane co-ordinates of these points are (−1, 0) and (−5, 0). Verify that, using the distance formula we get the same answer.

Remember this!

- Co-ordinates of origin are (0, 0). Hence if co-ordinates of point P are (x, y) then $d(O, P) = \sqrt{x^2+y^2}$.
- If points $P(x_1, y_1)$, $Q(x_2, y_2)$ lie on the XY plane then

  $d(P, Q) = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

  that is, $PQ^2 = (x_2-x_1)^2+(y_2-y_1)^2 = (x_1-x_2)^2+(y_1-y_2)^2$

Solved Examples

Ex. (1) Find the distance between the points P(−1, 1) and Q (5,−7) .

Solution :Suppose co-ordinates of point P are (x_1, y_1) and of point Q are (x_2, y_2).

$x_1 = -1, \quad y_1 = 1, \quad x_2 = 5, \quad y_2 = -7$

According to distance formula, $d(P, Q) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$$= \sqrt{[5-(-1)]^2 + [(-7)-1]^2}$$

$$= \sqrt{(6)^2 + (-8)^2}$$

$$= \sqrt{36+64}$$

$$d(P, Q) = \sqrt{100} = 10$$

∴ distance between points P and Q is 10.

Ex. (2) Show that points A(−3, 2), B(1, −2) and C(9, −10) are collinear.

Solution : If the sum of any two distances out of d(A, B), d(B, C) and d(A, C) is equal to the third , then the three points A, B and C are collinear.

∴ we will find d(A, B), d(B, C) and d(A, C).

Co-ordinates of A	Co-ordinates of B	Distance formula
(−3, 2)	(1, −2)	$d(A,B) = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
(x_1, y_1)	(x_2, y_2)	

$$\therefore d(A, B) = \sqrt{[1-(-3)]^2 + [(-2)-2]^2} \text{ .... from distance formula}$$

$$= \sqrt{(1+3)^2 + (-4)^2}$$

$$= \sqrt{16+16}$$

$$= \sqrt{32} = 4\sqrt{2} \quad \text{..........(I)}$$

$$d(B, C) = \sqrt{(9-1)^2 + (-10+2)^2}$$

$$= \sqrt{64+64} = 8\sqrt{2} \quad \text{..........(II)}$$

$$\text{and } d(A, C) = \sqrt{(9+3)^2 + (-10-2)^2}$$

$$= \sqrt{144+144} = 12\sqrt{2} \quad \text{..........(III)}$$

∴ from(I), (II) and (III) $4\sqrt{2} + 8\sqrt{2} = 12\sqrt{2}$

∴ d(A, B) + d(B, C) = d(A, C)

∴ Points A, B, C are collinear.

Ex. (3) Verify, whether points P(6, −6), Q(3, −7) and R(3, 3) are collinear.

Solution : $PQ = \sqrt{(6-3)^2+(-6+7)^2}$ ....by distance formula

$= \sqrt{(3)^2+(1)^2} = \sqrt{10}$ .... (I)

$QR = \sqrt{(3-3)^2+(-7-3)^2}$

$= \sqrt{(0)^2+(-10)^2} = \sqrt{100}$ .... (II)

$PR = \sqrt{(3-6)^2+(3+6)^2}$

$= \sqrt{(-3)^2+(9)^2} = \sqrt{90}$ .... (III)

From I, II and III out of $\sqrt{10}$, $\sqrt{100}$ and $\sqrt{90}$, $\sqrt{100}$ is the largest number.

Now we will verify whether $\left(\sqrt{100}\right)$ and $\left(\sqrt{10}+\sqrt{90}\right)$ are equal.

For this compare $\left(\sqrt{100}\right)^2$ and $\left(\sqrt{10}+\sqrt{90}\right)^2$.

You will see that $\left(\sqrt{10}+\sqrt{90}\right) > \left(\sqrt{100}\right)$ $\therefore$ PQ + PR ≠ QR

$\therefore$ points P(6, −6), Q(3, −7) and R(3, 3) are not collinear.

Ex. (4) Show that points (1, 7), (4, 2), (−1, −1) and (−4, 4) are vertices of a square.

Solution : In a quadrilateral, if all sides are of equal length and both diagonals are of equal length, then it is a square.

$\therefore$ we will find lengths of sides and diagonals by using the distance formula.

Suppose, A(1, 7), B(4, 2), C(−1, −1) and D(−4,4) are the given points.

$AB = \sqrt{(1-4)^2+(7-2)^2} = \sqrt{9+25} = \sqrt{34}$

$BC = \sqrt{(4+1)^2+(2+1)^2} = \sqrt{25+9} = \sqrt{34}$

$CD = \sqrt{(-1+4)^2+(-1-4)^2} = \sqrt{9+25} = \sqrt{34}$

$DA = \sqrt{(1+4)^2+(7-4)^2} = \sqrt{25+9} = \sqrt{34}$

$AC = \sqrt{(1+1)^2+(7+1)^2} = \sqrt{4+64} = \sqrt{68}$

$BD = \sqrt{(4+4)^2+(2-4)^2} = \sqrt{64+4} = \sqrt{68}$

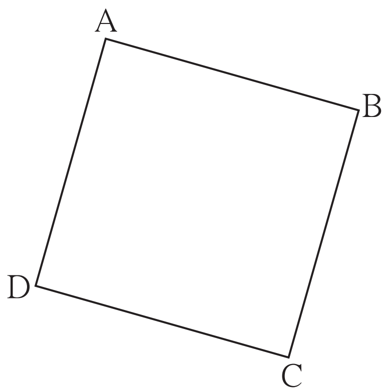


Fig. 5.9

$\therefore$ AB = BC = CD = DA and AC = BD

So, the lengths of four sides are equal and two diagonals are equal.

$\therefore$ (1,7), (4,2), (−1,−1) and (−4,4) are the vertices of a square.

Ex. (5) Find the co-ordinates of a point on Y- axis which is equidistant from M (−5,−2) and N(3,2)

Solution : Let point P(0, y) on Y- axis be equidistant from M(−5,−2) and N(3,2).

$$\therefore PM = PN \qquad \therefore PM^2 = PN^2$$

$$\therefore [0-(-5)]^2 + [y-(-2)]^2 = (0-3)^2 + (y-2)^2$$

$$\therefore 25 + (y+2)^2 = 9 + y^2 - 4y + 4$$

$$\therefore 25 + y^2 + 4y + 4 = 13 + y^2 - 4y$$

$$\therefore 8y = -16 \quad \therefore y = -2$$

$\therefore$ the co-ordinates of the point on the Y-axis which is equidistant from M and N are M (0, −2).

Ex. (6) A(−3, −4), B(−5, 0), C(3, 0) are the vertices of Δ ABC. Find the co-ordinates of the circumcentre of Δ ABC.

Solution : Let, P(a, b) be the circumcentre of Δ ABC.

$\therefore$ point P is equidistant from A,B and C.

$\therefore PA^2 = PB^2 = PC^2$ .... (I) $\qquad \therefore PA^2 = PB^2$

$$(a+3)^2 + (b+4)^2 = (a+5)^2 + (b-0)^2$$

$$\therefore a^2 + 6a + 9 + b^2 + 8b + 16 = a^2 + 10a + 25 + b^2$$

$$\therefore -4a + 8b = 0$$

$$\therefore a - 2b = 0 \text{ .... (II)}$$

Similarly $PA^2 = PC^2$ .... (I) From

$$\therefore (a+3)^2 + (b+4)^2 = (a-3)^2 + (b-0)^2$$

$$\therefore a^2 + 6a + 9 + b^2 + 8b + 16 = a^2 - 6a + 9 + b^2$$

$$\therefore 12a + 8b = -16$$

$$\therefore 3a + 2b = -4 \text{ .... (III)}$$

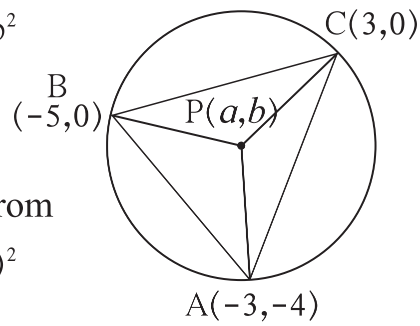


Fig. 5.10

Solving (II) and (III) we get $a = -1$, $b = -\frac{1}{2}$

$\therefore$ co-ordinates of circumcentre are $(-1, -\frac{1}{2})$.

Ex. (7) If point (x, y) is equidistant from points (7, 1) and (3, 5), show that $y = x-2$.

Solution : Let point P (x, y) be equidistant from points A(7, 1) and B(3, 5)

$$\therefore\ AP = BP$$
$$\therefore\ AP^2 = BP^2$$
$$\therefore\ (x-7)^2 + (y-1)^2 = (x-3)^2 + (y-5)^2$$
$$\therefore\ x^2 - 14x + 49 + y^2 - 2y + 1 = x^2 - 6x + 9 + y^2 - 10y + 25$$
$$\therefore -8x + 8y = -16$$
$$\therefore\ x - y = 2$$
$$\therefore\ y = x - 2$$

Ex. (8) Find the value of y if distance between points A (2,−2) and B (−1, y)is 5.

Solution : $AB^2 = [(-1) - 2]^2 + [y - (-2)]^2$ .... by distance formula

$\therefore\ 5^2 = (-3)^2 + (y+2)^2$

$\therefore\ 25 = 9 + (y+2)^2$

$\therefore\ 16 = (y+2)^2$

$\therefore\ y + 2 = \pm\sqrt{16}$

$\therefore\ y + 2 = \pm 4$

$\therefore\ y = 4 - 2$ or $y = -4 - 2$

$\therefore\ y = 2$ or $y = -6$

$\therefore$ value of y is 2 or −6.

Practice set 5.1

1. Find the distance between each of the following pairs of points.

   (1) A(2, 3), B(4, 1) (2) P(−5, 7), Q(−1, 3) (3) R(0, −3), S(0, $\frac{5}{2}$)

   (4) L(5, −8), M(−7, −3) (5) T(−3, 6), R(9, −10) (6) W($\frac{-7}{2}$, 4), X(11, 4)

2. Determine whether the points are collinear.

   (1) A(1, −3), B(2, −5), C(−4, 7) (2) L(−2, 3), M(1, −3), N(5, 4)

   (3) R(0, 3), D(2, 1), S(3, −1) (4) P(−2, 3), Q(1, 2), R(4, 1)

3. Find the point on the X-axis which is equidistant from A(−3, 4) and B(1, −4).

4. Verify that points P(−2, 2), Q(2, 2) and R(2, 7) are vertices of a right angled triangle.

5. Show that points P(2, −2), Q(7, 3), R(11, −1) and S (6, −6) are vertices of a parallelogram.

6. Show that points A(−4, −7), B(−1, 2), C(8, 5) and D(5, −4) are vertices of a rhombus ABCD.

7. Find x if distance between points L(x, 7) and M(1, 15) is 10.

8. Show that the points A(1, 2), B(1, 6), C($1 + 2\sqrt{3}$, 4) are vertices of an equilateral triangle.

Let's recall.

Property of intercepts made by three parallel lines :

In the figure line l || line m || line n,

line p and line q are transversals,

then $\frac{AB}{BC} = \frac{DE}{EF}$

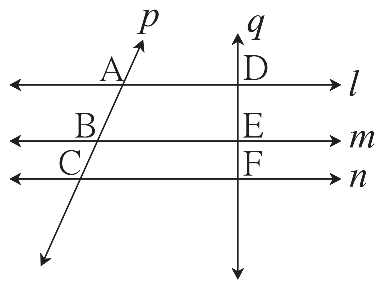


Fig. 5.11

Let's learn.

Division of a line segment

Fig. 5.12

In the figure, AP = 6 and PB = 10.

$\therefore \frac{AP}{PB} = \frac{6}{10} = \frac{3}{5}$

In other words it is said that 'point P divides the line segment AB in the ratio 3:5.

Let us see how to find the co-ordinates of a point on a segment which divides the segment in the given ratio.

Let's learn.

Section formula

In the figure 5.13, point P on the seg AB in XY plane, divides seg AB in the ratio $m : n$.

Assume $A(x_1, y_1)$ $B(x_2, y_2)$ and $P(x, y)$

Draw seg AC, seg PQ and seg BD perpendicular to X- axis.

$\therefore$ $C(x_1, 0)$; $Q(x, 0)$ and $D(x_2, 0)$.

$\therefore$ $\left.\begin{array}{l} CQ = x - x_1 \\ \text{and } QD = x_2 - x \end{array}\right\}$ .... (I)

seg AC || seg PQ || seg BD.

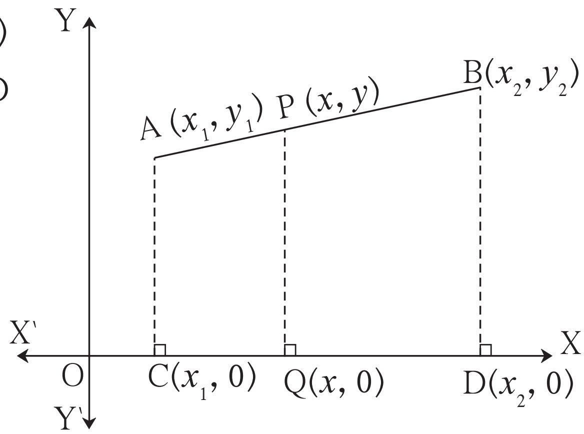


Fig. 5.13

$\therefore$ By the property of intercepts of three parallel lines, $\frac{AP}{PB} = \frac{CQ}{QD} = \frac{m}{n}$

Now $CQ = x - x_1$ and $QD = x_2 - x$ .... from (I)

$$\therefore \frac{x - x_1}{x_2 - x} = \frac{m}{n}$$

$$\therefore n(x - x_1) = m(x_2 - x)$$

$$\therefore nx - nx_1 = mx_2 - mx$$

$$\therefore mx + nx = mx_2 + nx_1$$

$$\therefore x(m + n) = mx_2 + nx_1$$

$$\therefore x = \frac{mx_2 + nx_1}{m + n}$$

Similarly drawing perpendiculars from points A, P and B to Y- axis,

we get, $y = \frac{my_2 + ny_1}{m + n}$.

$\therefore$ co-ordinates of the point, which divides the line segment joining the points $A(x_1, y_1)$ and $B(x_2, y_2)$ in the ratio $m : n$ are given by

$$\left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right).$$

Co-ordinates of the midpoint of a segment

If A(x_1, y_1)and B(x_2, y_2) are two points and P (x, y) is the midpoint of seg AB then $m = n$.

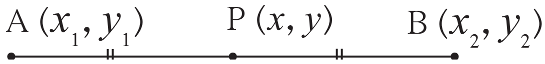


Fig. 5.14

$\therefore$ values of x and y can be written as

$$x = \frac{mx_2 + nx_1}{m+n}$$

$$= \frac{mx_2 + mx_1}{m+m} \quad \because \; m = n$$

$$= \frac{m(x_1 + x_2)}{2m}$$

$$= \frac{x_1 + x_2}{2}$$

$$y = \frac{my_2 + ny_1}{m+n}$$

$$= \frac{my_2 + my_1}{m+m} \quad \because \; m = n$$

$$= \frac{m(y_1 + y_2)}{2m}$$

$$= \frac{y_1 + y_2}{2}$$

$\therefore$ co-ordinates of midpoint P are $\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$

This is called as **midpoint formula**.

In the previous standard we have shown that $\frac{a+b}{2}$ is the midpoint of the segment joining two points indicating rational numbers a and b on a number line. Note that it is a special case of the above midpoint formula.

Solved Examples

Ex. (1) If A(3,5), B(7,9) and point Q divides seg AB in the ratio 2:3 then find co-ordinates of point Q.

Solution : In the given example let (x_1, y_1)= (3, 5)

and (x_2, y_2)= (7, 9) .

$m : n$ = 2:3

According to section formula,

$$x = \frac{mx_2 + nx_1}{m+n} = \frac{2\times7+3\times3}{2+3} = \frac{23}{5} \qquad y = \frac{my_2 + ny_1}{m+n} = \frac{2\times9+3\times5}{2+3} = \frac{33}{5}$$

$\therefore$ Co-ordinates of Q are $\left(\frac{23}{5}, \frac{33}{5}\right)$

Ex. (2) Find the co-ordinates of point P if P is the midpoint of a line segment AB with A(−4,2) and B(6,2).

Solution : In the given example, suppose

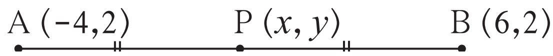


Fig. 5.15

$(-4, 2) = (x_1, y_1)$; $(6, 2) = (x_2, y_2)$ and co-ordinates of P are (x, y)

∴ according to midpoint theorem,

$$x = \frac{x_1 + x_2}{2} = \frac{-4+6}{2} = \frac{2}{2} = 1$$

$$y = \frac{y_1 + y_2}{2} = \frac{2+2}{2} = \frac{4}{2} = 2$$

∴ co-ordinates of midpoint P are (1,2) .

Let's recall.

We know that, medians of a triangle are concurrent .

The point of concurrence (centroid) divides the median in the ratio 2:1.

Let's learn.

Centroid formula

Suppose the co-ordinates of vertices of a triangle are given. Then we will find the co-ordinates of the centroid of the triangle.

In Δ ABC, $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$ are the vertices. Seg AD is a median and $G(x, y)$ is the centroid.

D is the mid point of line segment BC.

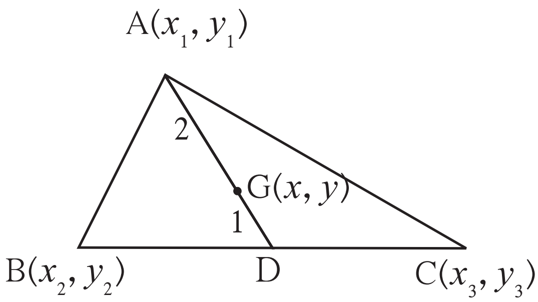


Fig. 5.16

$\therefore$ co-ordinates of point D are $x = \dfrac{x_2 + x_3}{2}$, $y = \dfrac{y_2 + y_3}{2}$ .... midpoint theorem

Point G(x, y) is centroid of triangle Δ ABC. $\therefore$ AG : GD = 2 : 1

$\therefore$ according to section formula,

$$x = \frac{2\left(\dfrac{x_2 + x_3}{2}\right) + 1 \times x_1}{2+1} = \frac{x_2 + x_3 + x_1}{3} = \frac{x_1 + x_2 + x_3}{3}$$

$$y = \frac{2\left(\dfrac{y_2 + y_3}{2}\right) + 1 \times y_1}{2+1} = \frac{y_2 + y_3 + y_1}{3} = \frac{y_1 + y_2 + y_3}{3}$$

Thus if (x_1, y_1), (x_2, y_2) and $(x_3, y_3)$are the vertices of a triangle then the co-ordinates of the centroid are $\left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$.

This is called the **centroid formula.**

Remember this!

- Section formula

  The co-ordinates of a point which divides the line segment joined by two distinct points (x_1, y_1) and (x_2, y_2) in the ratio $m : n$ are $\left(\dfrac{mx_2 + nx_1}{m+n}, \dfrac{my_2 + ny_1}{m+n}\right)$.

- Midpoint formula

  The co-ordinates of midpoint of a line segment joining two distinct points (x_1, y_1) and (x_2, y_2) are $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$.

- Centroid formula

  If (x_1, y_1), (x_2, y_2) and (x_3, y_3) are the vertices of a triangle then co-ordinates of the centroid are $\left(\dfrac{x_1 + x_2 + x_3}{3}, \dfrac{y_1 + y_2 + y_3}{3}\right)$.

Ex. (1) If point T divides the segment AB with A(-7,4) and B(-6,-5) in the ratio 7:2, find the co-ordinates of T.

Solution : Let the co-ordinates of T be (x, y).

$\therefore$ by the section formula,

$$x = \frac{mx_2 + nx_1}{m+n} = \frac{7\times(-6)+2\times(-7)}{7+2}$$

$$= \frac{-42-14}{9} = \frac{-56}{9}$$

$$y = \frac{my_2 + ny_1}{m+n} = \frac{7\times(-5)+2\times(4)}{7+2}$$

$$= \frac{-35+8}{9} = \frac{-27}{9} = -3$$

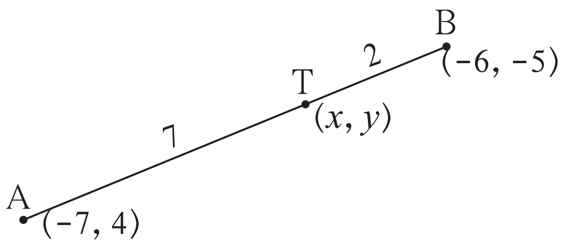


Fig. 5.17

$\therefore$ co-ordinates of point T are $\left(\frac{-56}{9}, -3\right)$.

Ex. (2) If point P(-4, 6) divides the line segment AB with A(-6, 10) and B(r, s) in the ratio 2:1, find the co-ordinates of B.

Solution : By section formula

$$-4 = \frac{2\times r + 1\times(-6)}{2+1}$$

$\therefore$ $-4 = \frac{2r-6}{3}$

$\therefore$ $-12 = 2r - 6$

$\therefore$ $2r = -6$

$\therefore$ $r = -3$

$$6 = \frac{2\times s + 1\times 10}{2+1}$$

$\therefore$ $6 = \frac{2s+10}{3}$

$\therefore$ $18 = 2s + 10$

$\therefore$ $2s = 8$

$\therefore$ $s = 4$

$\therefore$ co-ordinates of point B are (-3, 4).

Ex. (3) A(15,5), B(9,20) and A-P-B. Find the ratio in which point P(11,15) divides segment AB.

Solution : Suppose, point P(11,15) divides segment AB in the ratio $m : n$

$\therefore$ by section formula,

$x = \frac{mx_2 + nx_1}{m+n}$

$\therefore\ 11 = \frac{9m+15n}{m+n}$

$\therefore\ 11m + 11n = 9m + 15n$

$\therefore\ 2m = 4n$

$\therefore\ \frac{m}{n} = \frac{4}{2} = \frac{2}{1}$

$\therefore$ The required ratio is 2 : 1.

> Similarly, find the ratio using y co-ordinates. Write the conclusion.

Ex. (4) Find the co-ordinates of the points of trisection of the segment joining the points A (2,−2) and B(−7,4) .

(The two points that divide the line segment in three equal parts are called as points of trisection of the segment.)

Solution : Let points P and Q be the points of trisection of the line segment joining the points A and B.

Point P and Q divide line segment AB into three parts.

AP = PQ = QB .... (I)

$\frac{AP}{PB} = \frac{AP}{PQ+QB} = \frac{AP}{AP+AP} = \frac{AP}{2AP} = \frac{1}{2}$ .... From (I)

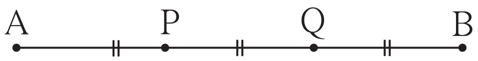


Fig. 5.18

Point P divides seg AB in the ratio 1:2.

x co-ordinate of point P = $\frac{1\times(-7)+2\times 2}{1+2} = \frac{-7+4}{3} = \frac{-3}{3} = -1$

y co-ordinate of point P = $\frac{1\times 4+2\times(-2)}{1+2} = \frac{4-4}{3} = \frac{0}{3} = 0$

Point Q divides seg AB in the ratio 2:1. $\therefore \frac{AQ}{QD} = \frac{2}{1}$

x co-ordinate of point Q = $\frac{2\times(-7)+1\times 2}{2+1} = \frac{-14+2}{3} = \frac{-12}{3} = -4$

y co-ordinate of point Q = $\frac{2\times 4+1\times -2}{2+1} = \frac{8-2}{3} = \frac{6}{3} = 2$

$\therefore$ co-ordinates of points of trisection are (−1, 0) and (−4, 2).

For more information :

See how the external division of the line segment joining points A and B takes place.

Let us see how the co-ordinates of point P can be found out if P divides the line segment joining points A(−4, 6) and B(5, 10) in the ratio 3:1 externally.

$\frac{AP}{PB} = \frac{3}{1}$ that is AP is larger than PB and A−B−P.

$\frac{AP}{PB} = \frac{3}{1}$ that is $AP = 3k$, $BP = k$, then $AB = 2k$

$$\therefore \frac{AB}{BP} = \frac{2}{1}$$

Now point B divides seg AP in the ratio 2 : 1.

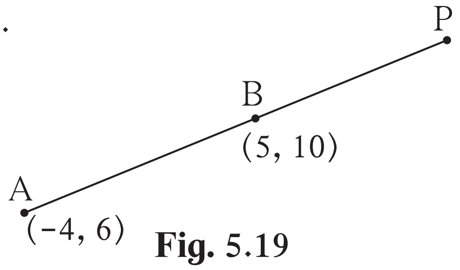


Fig. 5.19

We have learnt to find the coordinates of point P if co-ordinates of points A and B are known.

Practice set 5.2

1. Find the coordinates of point P if P divides the line segment joining the points A(−1,7) and B(4,−3) in the ratio 2 : 3.
2. In each of the following examples find the co-ordinates of point A which divides segment PQ in the ratio $a : b$.
   (1) P(−3, 7), Q(1, −4), $a : b = 2 : 1$
   (2) P(−2, −5), Q(4, 3), $a : b = 3 : 4$
   (3) P(2, 6), Q(−4, 1), $a : b = 1 : 2$
3. Find the ratio in which point T(−1, 6)divides the line segment joining the points P(−3, 10) and Q(6, −8).
4. Point P is the centre of the circle and AB is a diameter . Find the coordinates of point B if coordinates of point A and P are (2, −3) and (−2, 0) respectively.
5. Find the ratio in which point P(k, 7) divides the segment joining A(8, 9) and B(1, 2). Also find k .
6. Find the coordinates of midpoint of the segment joining the points (22, 20) and (0, 16).
7. Find the centroids of the triangles whose vertices are given below.
   (1)(−7, 6), (2, −2), (8, 5)
   (2) (3, −5), (4, 3), (11, −4)
   (3) (4, 7), (8, 4), (7, 11)

8. In Δ ABC, G (−4, −7) is the centroid. If A (−14, −19) and B(3, 5) then find the co-ordinates of C.

9. A(h, −6), B(2, 3) and C(−6, k) are the co-ordinates of vertices of a triangle whose centroid is G (1, 5). Find h and k.

10. Find the co-ordinates of the points of trisection of the line segment AB with A(2, 7) and B(−4, −8).

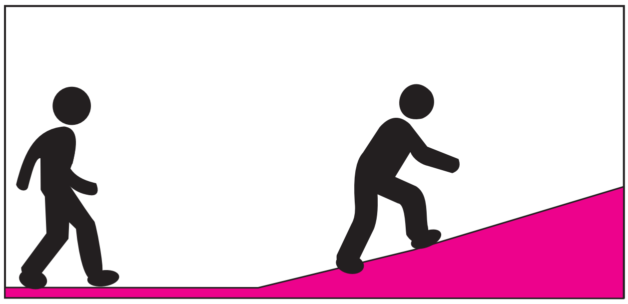
11. If A (−14, −10), B(6, −2) is given, find the coordinates of the points which divide segment AB into four equal parts.

12. If A (20, 10), B(0, 20) are given, find the coordinates of the points which divide segment AB into five congruent parts.

Let's learn.

Slope of a line

When we walk on a plane road we need not exert much effort but while climbing up a slope we need more effort. In science, we have studied that while climbing up a slope we have to work against gravitational force.



In co-ordinate geometry, slope of a line is an important concept. We will learn it through the following activity.

Activity I :

In the figure points A(−2, −5), B(0, −2), C(2, 1), D(4, 4), E(6, 7) lie on line l. Observe the table which is made with the help of coordinates of these points on line l.

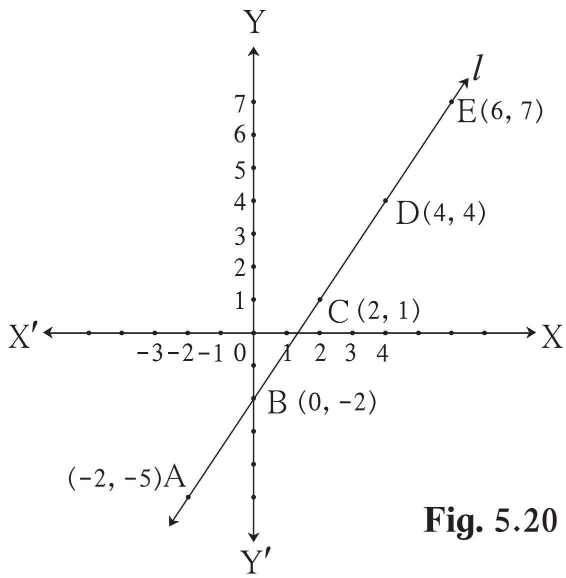


Fig. 5.20

Sr. No.	First point	Second point	Co-ordinates of first point (x_1, y_1)	Co-ordinates of second point (x_2, y_2)	$\frac{y_2 - y_1}{x_2 - x_1}$
1	C	E	(2, 1)	(6, 7)	$\frac{7-1}{6-2} = \frac{6}{4} = \frac{3}{2}$
2	A	D	(-2, -5)	(4, 4)	$\frac{4-(-5)}{4-(-2)} = \frac{9}{6} = \frac{3}{2}$
3	D	A	(4, 4)	(-2, -5)	$\frac{-5-4}{-2-4} = \frac{-9}{-6} = \frac{3}{2}$
4	B	C	--	--	--
5	C	A	--	--	--
6	A	C	--	--	--

Fill in the blank spaces in the above table. Similarly take some other pairs of points on line l and find the ratio $\frac{y_2 - y_1}{x_2 - x_1}$ for each pair.

From this activity, we understand that for any two points (x_1, y_1) and (x_2, y_2) on line l, the ratio $\frac{y_2 - y_1}{x_2 - x_1}$ is constant.

If (x_1, y_1) and (x_2, y_2) are any two points on line l, the ratio $\frac{y_2 - y_1}{x_2 - x_1}$ is called the slope of the line l.

Generally slope is shown by letter m.

$$\therefore \quad m = \frac{y_2 - y_1}{x_2 - x_1}$$

Activity II : In the figure, some points on line l, t and n are given. Find the slopes of those lines.

Now you will know,

(1) Slopes of line l and line t are positive.

(2) Slope of line n is negative.

(3) Slope of line t is more than slope of line l.

(4) Slopes of lines l and t which make acute angles with X- axis, are positive.

(5) Slope of line n making obtuse angle with X- axis is negative.

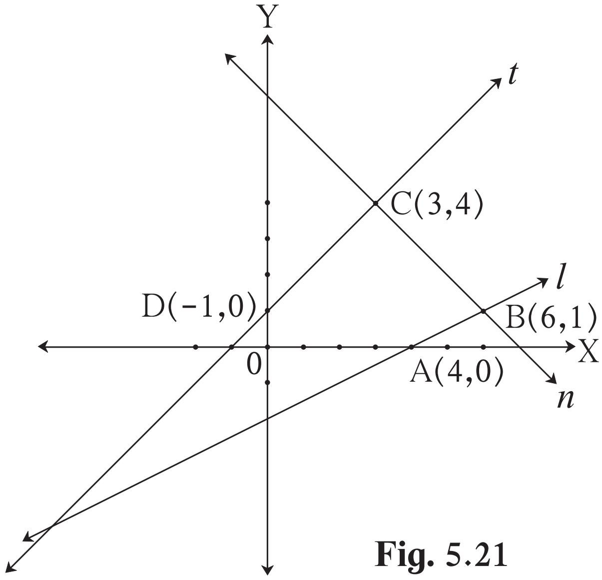


Fig. 5.21

Slopes of X-axis, Y-axis and lines parallel to axes.

In the figure 5.22, $(x_1, 0)$ and $(x_2, 0)$ are two points on the X- axis.

Slope of X- axis $= \dfrac{0 - 0}{x_2 - x_1} = 0$

In the same way $(0, y_1)$ and $(0, y_2)$ are two points on the Y- axis.

Slope of Y- axis $= \dfrac{y_2 - y_1}{0 - 0} = \dfrac{y_2 - y_1}{0}$,

But division by 0 is not possible.

$\therefore$ slope of Y- axis can not be determined.

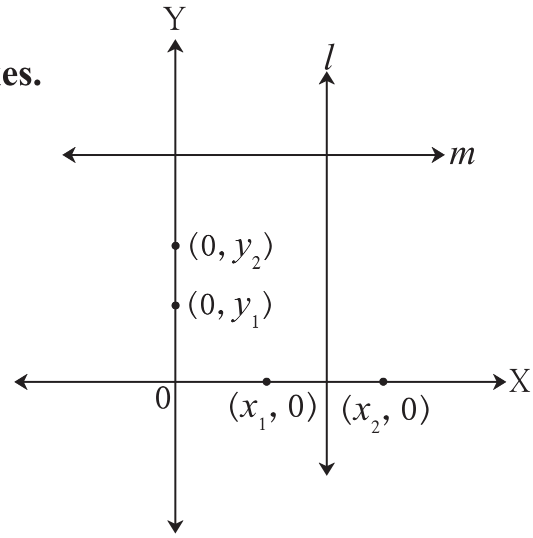


Fig. 5.22

Now try to find the slope of any line like line m which is parallel to X- axis. It will come out to be 0.

Similarly we cannot determine the slope of a line like line l which is parallel to Y- axis

Slope of line – using ratio in trigonometry

In the figure 5.23, points $P(x_1, y_1)$ and $Q(x_2, y_2)$ are on line l.

Line l intersects X axis in point T.

seg QS $\perp$ X- axis, seg PR $\perp$ seg QS $\therefore$ seg PR $\parallel$ seg TS .... corresponding angle test

$\therefore$ QR $= y_2 - y_1$ and PR $= x_2 - x_1$

$\therefore \quad \frac{QR}{PR} = \frac{y_2 - y_1}{x_2 - x_1}$ .... (I)

Line TQ makes an angle θ with the X- axis .

$\therefore \quad \frac{QR}{PR} = \tan\theta$ .... (II)

$\therefore$ From (I) and (II), $\frac{y_2 - y_1}{x_2 - x_1} = \tan\theta$

$\therefore \quad m = \tan\theta$

Now seg PR || seg TS, line l is transversal

$\therefore \quad \angle QPR = \angle QTS$ ......corresponding angles

From this, we can define slope in another way. The tan ratio of an angle made by the line with the positive direction of X-axis is called the slope of that line.

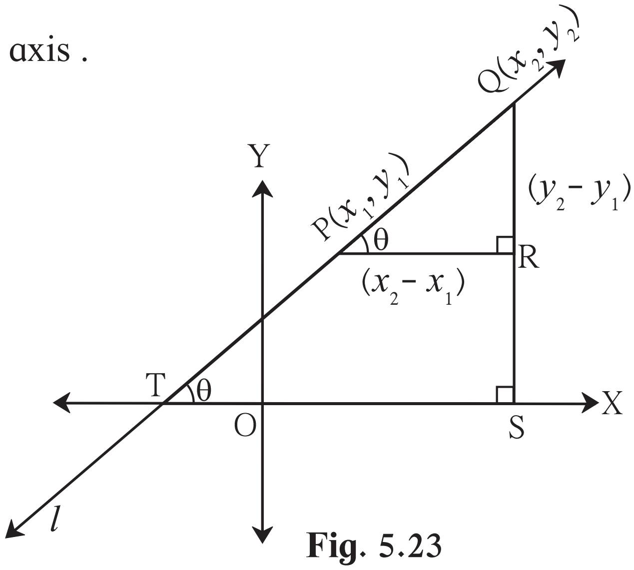


Fig. 5.23

When any two lines have same slope, these lines make equal angles with the positive direction of X- axis.

$\therefore$ These two lines are parallel.

Slope of Parallel Lines

Activity :

In the figure 5.22 both line l and line t make angle θ with the positive direction of X- axis.

$\therefore$ line l || line t .... corresponding angle test

Consider, point A(-3, 0) and point B(0, 3) on line l

Find the slope of line l.

Slope of line $l = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{\square - \square}{\square - \square} = \frac{\square}{\square}$

$= \square$

In the similar way, consider suitable points on the line t and find the slope of line t.

From this, you can verify that parallel lines have equal slopes.

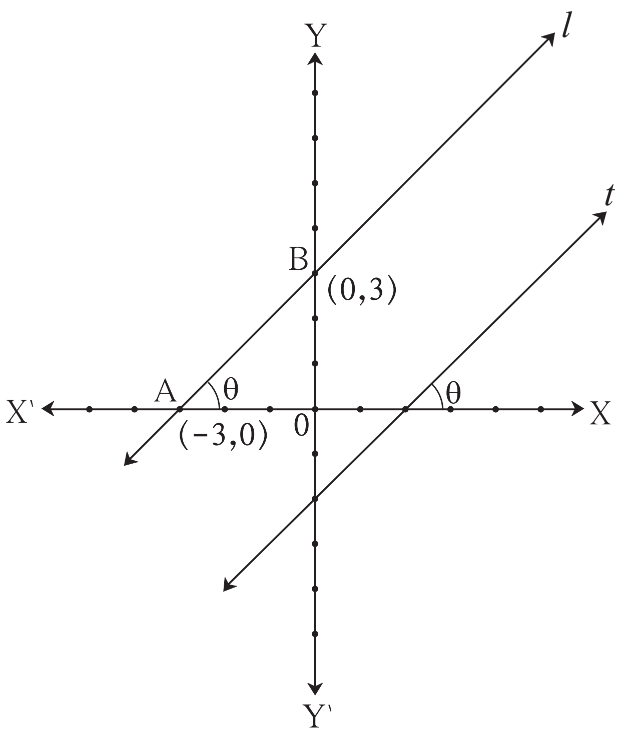


Fig. 5.24

Here $\theta = 45°$.

Use slope, $m = \tan\theta$ and verify that slopes of parallel lines are equal.

Similarly taking $\theta = 30°$, $\theta = 60°$ verify that slopes of parallel lines are equal.

Remember this!

The slope of X- axis and of any line parallel to X- axis is zero.

The slope of Y- axis and of any line parallel to Y- axis cannot be determined.

Solved Examples

EX. (1) Find the slope of the line passing through the points A $(-3, 5)$, and B $(4, -1)$

Solution : Let, $x_1 = -3$, $x_2 = 4$, $y_1 = 5$, $y_2 = -1$

$\therefore$ Slope of line AB $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{-1-5}{4-(-3)} = \dfrac{-6}{7}$

EX. (2) Show that points $P(-2, 3)$, $Q(1, 2)$, $R(4, 1)$ are collinear.

Solution : $P(-2, 3)$, $Q(1, 2)$ and $R(4, 1)$ are given points

slope of line PQ $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{2-3}{1-(-2)} = -\dfrac{1}{3}$

Slope of line QR $= \dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{1-2}{4-1} = -\dfrac{1}{3}$

Slope of line PQ and line QR is equal.

But point Q lies on both the lines.

$\therefore$ Point P, Q, R are collinear.

EX. (3) If slope of the line joining points $P(k, 0)$ and $Q(-3, -2)$ is $\dfrac{2}{7}$ then find k.

Solution : $P(k, 0)$ and $Q(-3, -2)$

Slope of line PQ $= \dfrac{-2-0}{-3-k} = \dfrac{-2}{-3-k}$

But slope of line PQ is given to be $\dfrac{2}{7}$.

$\therefore \dfrac{-2}{-3-k} = \dfrac{2}{7}$ $\qquad \therefore k = 4$

EX. (4) If A (6, 1), B (8, 2), C (9, 4) and D (7, 3) are the vertices of □ ABCD, show that □ ABCD is a parallelogram.

Solution : You know that Slope of line = $\frac{y_2 - y_1}{x_2 - x_1}$

Slope of line AB $= \frac{2-1}{8-6} = \frac{1}{2}$ .... (I)

Slope of line BC $= \frac{4-2}{9-8} = 2$ .... (II)

Slope of line CD $= \frac{3-4}{7-9} = \frac{1}{2}$ .... (III)

Slope of line DA $= \frac{3-1}{7-6} = 2$ .... (IV)

Slope of line AB = Slope of line CD .... From (I) and (III)

$\therefore$ line AB || line CD

Slope of line BC = Slope of line DA .... From (II) and (IV)

$\therefore$ line BC || line DA

Both the pairs of opposite sides of the quadrilateral are parallel

$\therefore$ □ ABCD is a parallelogram.

Practice set 5.3

1. Angles made by the line with the positive direction of X-axis are given. Find the slope of these lines.

(1) 45° (2) 60° (3) 90°

2. Find the slopes of the lines passing through the given points.

(1) A (2, 3), B (4, 7) (2) P (–3, 1), Q (5, –2)

(3) C (5, –2), D (7, 3) (4) L (–2, –3), M (–6, –8)

(5) E(–4, –2), F (6, 3) (6) T (0, –3), S (0, 4)

3. Determine whether the following points are collinear.

(1) A(–1, –1), B(0, 1), C(1, 3) (2) D(–2, –3), E(1, 0), F(2, 1)

(3) L(2, 5), M(3, 3), N(5, 1) (4) P(2, –5), Q(1, –3), R(–2, 3)

(5) R(1, –4), S(–2, 2), T(–3, 4) (6) A(–4, 4), K$(-2, \frac{5}{2})$, N(4, –2)

4. If A (1, –1), B (0, 4), C (–5, 3) are vertices of a triangle then find the slope of each side.

5. Show that A (–4, –7), B (–1, 2), C (8, 5) and D (5, –4) are the vertices of a parallelogram.

6. Find k, if R$(1, -1)$, S $(-2, k)$ and slope of line RS is -2.

7. Find k, if B$(k, -5)$, C $(1, 2)$ and slope of the line is 7.

8. Find k, if PQ || RS and P$(2, 4)$, Q $(3, 6)$, R$(3, 1)$, S$(5, k)$.

Problem set 5

1. Fill in the blanks using correct alternatives.

(1) Seg AB is parallel to Y-axis and coordinates of point A are (1,3) then co-ordinates of point B can be ....

(A) (3,1) (B) (5,3) (C) (3,0) (D) (1,-3)

(2) Out of the following, point .... lies to the right of the origin on X- axis.

(A) (-2,0) (B) (0,2) (C) (2,3) (D) (2,0)

(3) Distance of point (-3,4) from the origin is ....

(A) 7 (B) 1 (C) 5 (D) -5

(4) A line makes an angle of $30°$ with the positive direction of X- axis. So the slope of the line is ....

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{\sqrt{3}}$ (D) $\sqrt{3}$

2. Determine whether the given points are collinear.

(1) A(0,2) , B(1,-0.5), C(2,-3)

(2) P(1, 2) , Q$(2, \frac{8}{5})$, R$(3, \frac{6}{5})$

(3) L(1,2) , M(5,3) , N(8,6)

3. Find the coordinates of the midpoint of the line segment joining P(0,6) and Q(12,20).

4. Find the ratio in which the line segment joining the points A(3,8) and B(-9, 3) is divided by the Y- axis.

5. Find the point on X-axis which is equidistant from P(2,-5) and Q(-2,9).

6. Find the distances between the following points.

(i) A$(a, 0)$, B$(0, a)$ (ii) P(-6, -3), Q(-1, 9) (iii) R$(-3a, a)$, S$(a, -2a)$

7. Find the coordinates of the circumcentre of a triangle whose vertices are (-3,1), (0,-2) and (1,3)

8. In the following examples, can the segment joining the given points form a triangle ? If triangle is formed, state the type of the triangle considering sides of the triangle.
   (1) L(6,4) , M(−5,−3) , N(−6,8)
   (2) P(−2,−6) , Q(−4,−2), R(−5,0)
   (3) $A(\sqrt{2}, \sqrt{2})$, $B(-\sqrt{2}, -\sqrt{2})$, $C(-\sqrt{6}, \sqrt{6})$

9. Find k if the line passing through points P(−12,−3) and Q(4, k)has slope $\frac{1}{2}$.

10. Show that the line joining the points A(4, 8) and B(5, 5) is parallel to the line joining the points C(2,4) and D(1,7).

11. Show that points P(1,−2), Q(5,2), R(3,−1), S(−1,−5) are the vertices of a parallelogram

12. Show that the □ PQRS formed by P(2,1), Q(−1,3), R(−5,−3) and S(−2,−5) is a rectangle

13. Find the lengths of the medians of a triangle whose vertices are A(−1, 1), B(5, −3) and C(3, 5) .

14*. Find the coordinates of centroid of the triangles if points D(−7, 6), E(8, 5) and F(2, −2) are the mid points of the sides of that triangle.

15. Show that A(4, −1), B(6, 0), C(7, −2) and D(5, −3)are vertices of a square.

16. Find the coordinates of circumcentre and radius of circumcircle of Δ ABC if A(7, 1), B(3, 5) and C(2, 0) are given.

17. Given A(4,−3), B(8,5). Find the coordinates of the point that divides segment AB in the ratio 3:1.

18*. Find the type of the quadrilateral if points A(−4, −2), B(−3, −7) C(3, −2) and D(2, 3) are joined serially.

19*. The line segment AB is divided into five congruent parts at P, Q, R and S such that A−P−Q−R−S−B. If point Q(12, 14) and S(4, 18) are given find the coordinates of A, P, R,B.

20. Find the coordinates of the centre of the circle passing through the points P(6,−6), Q(3,−7)and R(3,3).

21*. Find the possible pairs of coordinates of the fourth vertex D of the parallelogram, if three of its vertices are A(5,6), B(1,−2)and C(3,−2).

22. Find the slope of the diagonals of a quadrilateral with vertices A(1,7), B(6,3), C(0,−3) and D(−3,3).

PUX2HU