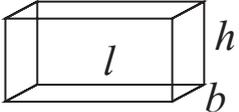
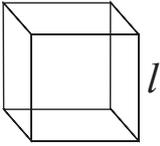
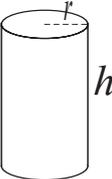
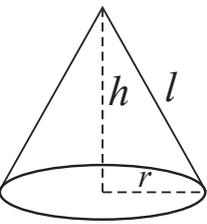


**Let's study.**

- Mixed examples on surface area and volume of different solid figures
- Arc of circle - length of arc
- Area of a sector
- Area of segment of a circle

**Let's recall.**

Last year we have studied surface area and volume of some three dimensional figures. Let us recall the formulae to find the surface areas and volumes.

No.	Three dimensional figure	Formulae
1 .	Cuboid 	Lateral surface area = $2h(l + b)$ Total surface area = $2(lb + bh + hl)$ Volume = lbh
2 .	Cube 	Lateral surface area = $4l^2$ Total surface area = $6l^2$ Volume = l^3
3 .	Cylinder 	Curved surface area = $2\pi rh$ Total surface area = $2\pi r(r + h)$ Volume = $\pi r^2 h$
4 .	Cone 	Slant height (l) = $\sqrt{h^2 + r^2}$ Curved surface area = πrl Total surface area = $\pi r(r + l)$ Volume = $\frac{1}{3} \times \pi r^2 h$

Practice set 7.1

1. Find the volume of a cone if the radius of its base is 1.5 cm and its perpendicular height is 5 cm.
2. Find the volume of a sphere of diameter 6 cm.
3. Find the total surface area of a cylinder if the radius of its base is 5 cm and height is 40 cm.
4. Find the surface area of a sphere of radius 7 cm.
5. The dimensions of a cuboid are 44 cm, 21 cm, 12 cm. It is melted and a cone of height 24 cm is made. Find the radius of its base.
- 6.

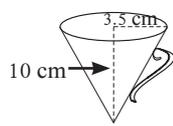


Fig 7.8
conical water jug

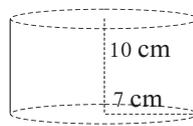


Fig 7.9
cylindrical water pot

Observe the measures of pots in figure 7.8 and 7.9. How many jugs of water can the cylindrical pot hold?

7. A cylinder and a cone have equal bases. The height of the cylinder is 3 cm and the area of its base is 100 cm^2 . The cone is placed upon the cylinder. Volume of the solid figure so formed is 500 cm^3 . Find the total height of the figure.

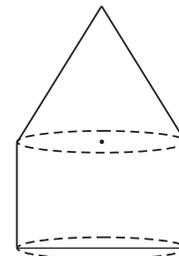


Fig 7.10

8. In figure 7.11, a toy made from a hemisphere, a cylinder and a cone is shown. Find the total area of the toy.



Fig. 7.11

9. In the figure 7.12, a cylindrical wrapper of flat tablets is shown. The radius of a tablet is 7 mm and its thickness is 5 mm. How many such tablets are wrapped in the wrapper?



Fig. 7.12

10. Figure 7.13 shows a toy. Its lower part is a hemisphere and the upper part is a cone. Find the volume and the surface area of the toy from the measures shown in the figure. ($\pi = 3.14$)

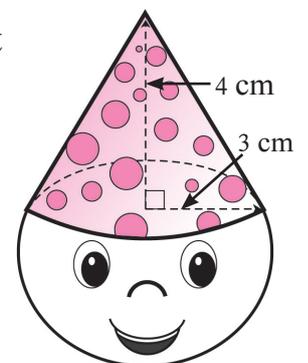


Fig. 7.13

11. Find the surface area and the volume of a beach ball shown in the figure.

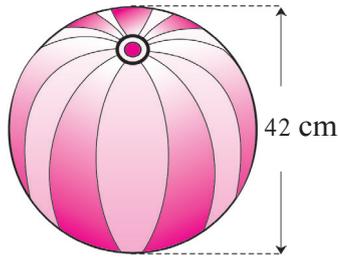


Fig. 7.14

12. As shown in the figure, a cylindrical glass contains water. A metal sphere of diameter 2 cm is immersed in it. Find the volume of the water.

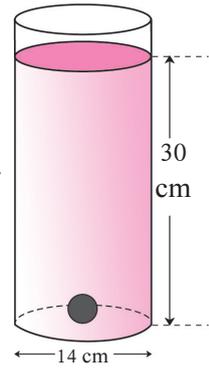


Fig. 7.15



Frustum of a cone

The shape of glass used to drink water as well as the shape of water it contains, are examples of frustum of a cone.

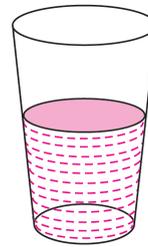


Fig. 7.16

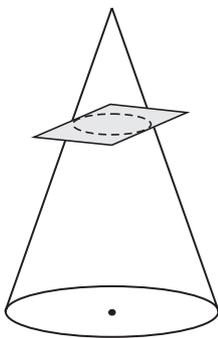


Fig. 7.17

A cone being cut

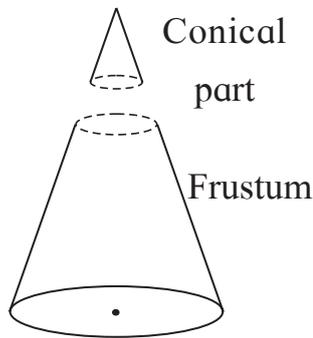


Fig. 7.18

Two parts of the cone

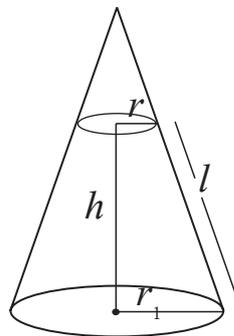


Fig. 7.19

Frustum

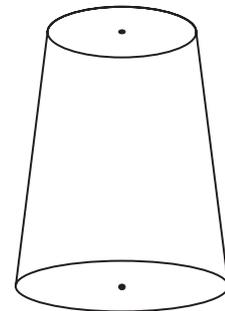


Fig. 7.20

A glass placed upside down

When a cone is cut parallel to its base we get two figures; one is a cone and the other is a frustum.

Volume and surface area of a frustum can be calculated by the formulae given below.



Remember this!

h = height of a frustum, l = slant height of a frustum,
 r_1 and r_2 = radii of circular faces of a frustum ($r_1 > r_2$)
 Slant height of a frustum $= l = \sqrt{h^2 + (r_1 - r_2)^2}$
 Curved surface area of a frustum $= \pi l (r_1 + r_2)$
 Total surface area of a frustum $= \pi l (r_1 + r_2) + \pi r_1^2 + \pi r_2^2$
 Volume of a frustum $= \frac{1}{3} \pi h (r_1^2 + r_2^2 + r_1 \times r_2)$

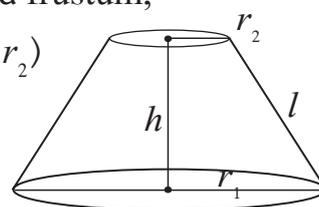


Fig. 7.21

Solved Examples

Ex. (1) A bucket is frustum shaped. Its height is 28 cm. Radii of circular faces are 12 cm and 15 cm. Find the capacity of the bucket. ($\pi = \frac{22}{7}$)

Solution : $r_1 = 15$ cm, $r_2 = 12$ cm, $h = 28$ cm

Capacity of the bucket = Volume of frustum

$$\begin{aligned}
 &= \frac{1}{3} \pi h (r_1^2 + r_2^2 + r_1 \times r_2) \\
 &= \frac{1}{3} \times \frac{22}{7} \times 28 (15^2 + 12^2 + 15 \times 12) \\
 &= \frac{22 \times 4}{3} \times (225 + 144 + 180) \\
 &= \frac{22 \times 4}{3} \times 549 \\
 &= 88 \times 183 \\
 &= 16104 \text{ cm}^3 = 16.104 \text{ litre}
 \end{aligned}$$

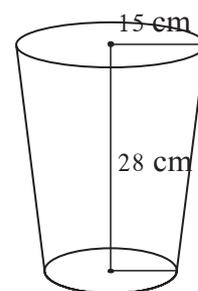


Fig. 7.22

\therefore capacity of the bucket is 16.104 litre.

Ex. (2) Radii of the top and the base of a frustum are 14 cm, 8 cm respectively. Its height is 8 cm. Find its

- i) curved surface area ii) total surface area iii) volume.

Solution : $r_1 = 14$ cm, $r_2 = 8$ cm, $h = 8$ cm

$$\begin{aligned}
 \text{Slant height of the frustum} = l &= \sqrt{h^2 + (r_1 - r_2)^2} \\
 &= \sqrt{8^2 + (14 - 8)^2} \\
 &= \sqrt{64 + 36} = 10 \text{ cm}
 \end{aligned}$$

$$\begin{aligned} \text{Curved surface area of the frustum} &= \pi(r_1 + r_2) l \\ &= 3.14 \times (14 + 8) \times 10 \\ &= 690.8 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Total surface area of frustum} &= \pi l (r_1 + r_2) + \pi r_1^2 + \pi r_2^2 \\ &= 3.14 \times 10 (14 + 8) + 3.14 \times 14^2 + 3.14 \times 8^2 \\ &= 690.8 + 615.44 + 200.96 \\ &= 690.8 + 816.4 \\ &= 1507.2 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Volume of the frustum} &= \frac{1}{3} \pi h (r_1^2 + r_2^2 + r_1 \times r_2) \\ &= \frac{1}{3} \times 3.14 \times 8 (14^2 + 8^2 + 14 \times 8) \\ &= 3114.88 \text{ cm}^3 \end{aligned}$$

Practice set 7.2

1. The radii of two circular ends of frustum shape bucket are 14 cm and 7 cm. Height of the bucket is 30 cm. How many liters of water it can hold ?
(1 litre = 1000 cm³)
2. The radii of ends of a frustum are 14 cm and 6 cm respectively and its height is 6 cm. Find its
i) curved surface area ii) total surface area. iii) volume ($\pi = 3.14$)
3. The circumferences of circular faces of a frustum are 132 cm and 88 cm and its height is 24 cm. To find the curved surface area of the frustum complete the following activity. ($\pi = \frac{22}{7}$).

circumference₁ = $2\pi r_1 = 132$

$$r_1 = \frac{132}{2\pi} = \boxed{}$$

circumference₂ = $2\pi r_2 = 88$

$$r_2 = \frac{88}{2\pi} = \boxed{}$$

slant height of frustum, $l = \sqrt{h^2 + (r_1 - r_2)^2}$

$$= \sqrt{\boxed{}^2 + \boxed{}^2}$$

$$= \boxed{} \text{ cm}$$

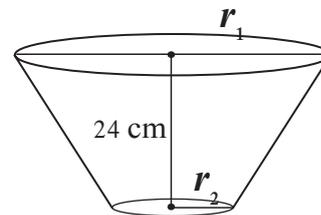


Fig. 7.23

$$\begin{aligned} \text{curved surface area of the frustum} &= \pi(r_1 + r_2)l \\ &= \pi \times \boxed{} \times \boxed{} \\ &= \boxed{} \text{ sq.cm.} \end{aligned}$$



Complete the following table with the help of figure 7.24.

Type of arc	Name of the arc	Measure of the arc
Minor arc	arc AXB
.....	arc AYB

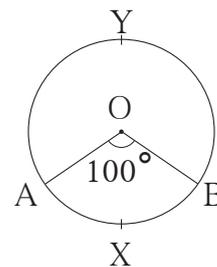


Fig. 7.24



Sector of a circle

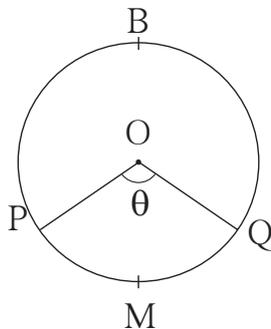


Fig. 7.25

In the adjacent figure, the central angle divides the circular region in two parts. Each of the parts is called a sector of the circle. Sector of a circle is the part enclosed by two radii of the circle and the arc joining their end points.

In the figure 7.25, O-PMQ and O-PBQ are two sectors of the circle.

Minor Sector :

Sector of a circle enclosed by two radii and their corresponding minor arc is called a ‘minor sector’.

In the above figure O-PMQ is a minor sector.

Major Sector :

Sector of a circle that is enclosed by two radii and their corresponding major arc is called a ‘major sector’.

In the above figure, O-PBQ is a major sector.

A relation between length of an arc and area of the sector

$$\text{Area of a sector, (A)} = \frac{\theta}{360} \times \pi r^2 \dots\dots\dots \text{I}$$

$$\text{Length of an arc, (l)} = \frac{\theta}{360} \times 2\pi r$$

$$\therefore \frac{\theta}{360} = \frac{l}{2\pi r} \dots\dots\dots \text{II}$$

$$\therefore A = \frac{l}{2\pi r} \times \pi r^2 \dots\dots\dots \text{From I and II}$$

$$A = \frac{1}{2} lr = \frac{lr}{2}$$

$$\therefore \text{Area of a sector} = \frac{\text{Length of the arc} \times \text{Radius}}{2}$$

$$\text{Similarly, } \frac{A}{\pi r^2} = \frac{l}{2\pi r} = \frac{\theta}{360}$$

~~~~~ Solved Examples ~~~~~

Ex. (1) The measure of a central angle of a circle is 150° and radius of the circle is 21 cm. Find the length of the arc and area of the sector associated with the central angle.

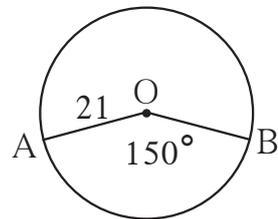


Fig. 7.28

Solution : $r = 21$ cm, $\theta = 150$, $\pi = \frac{22}{7}$

$$\begin{aligned} \text{Area of the sector, A} &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{150}{360} \times \frac{22}{7} \times 21 \times 21 \\ &= \frac{1155}{2} = 577.5 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} \text{Length of the arc, l} &= \frac{\theta}{360} \times 2\pi r \\ &= \frac{150}{360} \times 2 \times \frac{22}{7} \times 21 \\ &= 55 \text{ cm} \end{aligned}$$

Activity In figure 7.30, side of square ABCD is 7 cm. With centre D and radius DA, sector D - AXC is drawn. Fill in the following boxes properly and find out the area of the shaded region.

Solution : Area of a square = (Formula)
 =
 = 49 cm²

Area of sector (D- AXC) = (Formula)
 = × $\frac{22}{7}$ ×
 = 38.5 cm²

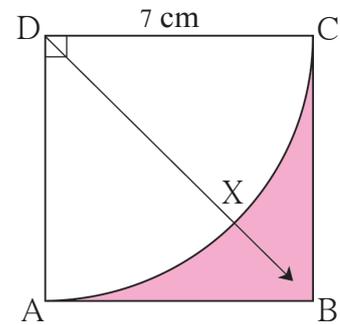


Fig. 7.30

A (shaded region) = A - A
 = cm² - cm²
 = cm²

Practice set 7.3

1. Radius of a circle is 10 cm. Measure of an arc of the circle is 54°. Find the area of the sector associated with the arc. ($\pi = 3.14$)
2. Measure of an arc of a circle is 80 cm and its radius is 18 cm. Find the length of the arc. ($\pi = 3.14$)
3. Radius of a sector of a circle is 3.5 cm and length of its arc is 2.2 cm. Find the area of the sector.
4. Radius of a circle is 10 cm. Area of a sector of the circle is 100 cm². Find the area of its corresponding major sector. ($\pi = 3.14$)
5. Area of a sector of a circle of radius 15 cm is 30 cm². Find the length of the arc of the sector.

6. In the figure 7.31, radius of the circle is 7 cm and $m(\text{arc MBN}) = 60^\circ$, find (1) Area of the circle .

- (2) A(O - MBN) .
- (3) A(O - MCN) .

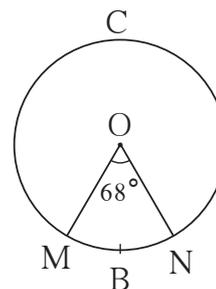


Fig. 7.31

$$PT = r \times \sin\theta \quad (\because OP = r)$$

$$\begin{aligned} A(\Delta OPQ) &= \frac{1}{2} \times \text{base} \times \text{height} \\ &= \frac{1}{2} \times OQ \times PT \\ &= \frac{1}{2} \times r \times r \sin\theta \\ &= \frac{1}{2} \times r^2 \sin\theta \dots\dots\dots (II) \end{aligned}$$

From (I) and (II) ,

$$\begin{aligned} A(\text{segment PXQ}) &= \frac{\theta}{360} \times \pi r^2 - \frac{1}{2} r^2 \times \sin\theta \\ &= r^2 \left[\frac{\pi\theta}{360} - \frac{\sin\theta}{2} \right] \end{aligned}$$

(Note that, we have studied the sine ratios of acute angles only. So we can use the above formula when $\theta \leq 90^\circ$.)

Solved Examples

Ex. (1) In the figure 7.40, $\angle AOB = 30^\circ$,
 $OA = 12 \text{ cm}$. Find the area of
the segment. ($\pi = 3.14$)

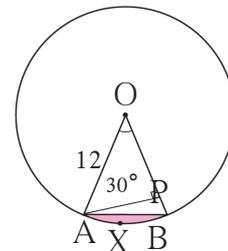


Fig. 7.40

Method I

$$\begin{aligned} r &= 12, \quad \theta = 30^\circ, \quad \pi = 3.14 \\ A(O\text{-}AXB) &= \frac{\theta}{360} \times \pi r^2 \\ &= \frac{30}{360} \times 3.14 \times 12^2 \\ &= 3.14 \times 12 \\ &= 37.68 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned} A(\Delta OAB) &= \frac{1}{2} r^2 \times \sin\theta \\ &= \frac{1}{2} \times 12^2 \times \sin 30 \\ &= \frac{1}{2} \times 144 \times \frac{1}{2} \\ &\dots\dots(\because \sin 30^\circ = \frac{1}{2}) \\ &= 36 \text{ cm}^2 \end{aligned}$$

$$\begin{aligned}
 A(\text{major segment}) &= A(\text{circle}) - A(\text{minor segment}) \\
 &= 3.14 \times 10^2 - 28.5 \\
 &= 314 - 28.5 \\
 &= 285.5 \text{ cm}^2
 \end{aligned}$$

Ex. (3) A regular hexagon is inscribed in a circle of radius 14 cm. Find the area of the region between the circle and the hexagon. $(\pi = \frac{22}{7}, \sqrt{3} = 1.732)$

Solution : side of the hexagon = 14 cm

$$\begin{aligned}
 A(\text{hexagon}) &= 6 \times \frac{\sqrt{3}}{4} \times (\text{side})^2 \\
 &= 6 \times \frac{\sqrt{3}}{4} \times 14^2 \\
 &= 509.208 \text{ cm}^2
 \end{aligned}$$

$$\begin{aligned}
 A(\text{circle}) &= \pi r^2 \\
 &= \frac{22}{7} \times 14 \times 14 \\
 &= 616 \text{ cm}^2
 \end{aligned}$$

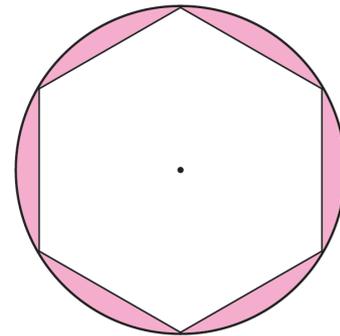


Fig. 7.42

The area of the region between the circle and the hexagon

$$\begin{aligned}
 &= A(\text{circle}) - A(\text{hexagon}) \\
 &= 616 - 509.208 \\
 &= 106.792 \text{ cm}^2
 \end{aligned}$$

Practice set 7.4

1. In figure 7.43, A is the centre of the circle. $\angle ABC = 45^\circ$ and $AC = 7\sqrt{2}$ cm. Find the area of segment BXC.

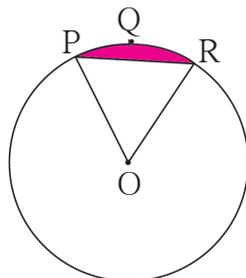


Fig. 7.44

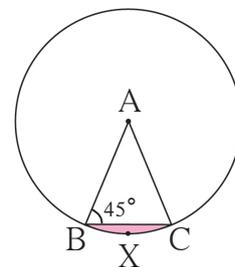


Fig. 7.43

2. In the figure 7.44, O is the centre of the circle. $m(\text{arc PQR}) = 60^\circ$ $OP = 10$ cm. Find the area of the shaded region. $(\pi = 3.14, \sqrt{3} = 1.73)$

3. In the figure 7.45, if A is the centre of the circle. $\angle PAR = 30^\circ$, $AP = 7.5$, find the area of the segment PQR

$(\pi = 3.14)$

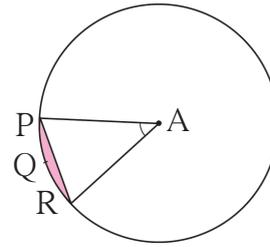


Fig. 7.45

- 4.

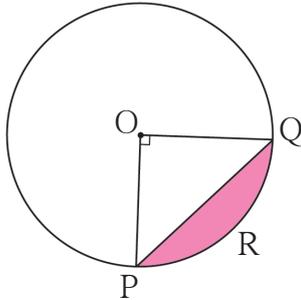


Fig. 7.46

In the figure 7.46, if O is the centre of the circle, PQ is a chord. $\angle POQ = 90^\circ$, area of shaded region is 114 cm^2 , find the radius of the circle. $(\pi = 3.14)$

5. A chord PQ of a circle with radius 15 cm subtends an angle of 60° with the centre of the circle. Find the area of the minor as well as the major segment.

$(\pi = 3.14, \sqrt{3} = 1.73)$

Problem set 7

1. Choose the correct alternative answer for each of the following questions.
- (1) The ratio of circumference and area of a circle is 2:7. Find its circumference.
 (A) 14π (B) $\frac{7}{\pi}$ (C) 7π (D) $\frac{14}{\pi}$
 - (2) If measure of an arc of a circle is 160° and its length is 44 cm, find the circumference of the circle.
 (A) 66 cm (B) 44 cm (C) 160 cm (D) 99 cm
 - (3) Find the perimeter of a sector of a circle if its measure is 90° and radius is 7 cm.
 (A) 44 cm (B) 25 cm (C) 36 cm (D) 56 cm
 - (4) Find the curved surface area of a cone of radius 7 cm and height 24 cm.
 (A) 440 cm^2 (B) 550 cm^2 (C) 330 cm^2 (D) 110 cm^2
 - (5) The curved surface area of a cylinder is 440 cm^2 and its radius is 5 cm. Find its height.
 (A) $\frac{44}{\pi} \text{ cm}$ (B) $22\pi \text{ cm}$ (C) $44\pi \text{ cm}$ (D) $\frac{22}{\pi} \text{ cm}$
 - (6) A cone was melted and cast into a cylinder of the same radius as that of the base of the cone. If the height of the cylinder is 5 cm, find the height of the cone.
 (A) 15 cm (B) 10 cm (C) 18 cm (D) 5 cm

11. In the figure 7.48, square ABCD is inscribed in the sector A-PCQ. The radius of sector C-BXD is 20 cm. Complete the following activity to find the area of shaded region.

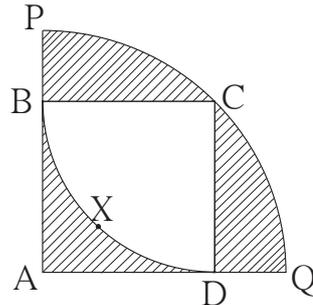


Fig. 7.48

Solution : Side of square ABCD = radius of sector C - BXD = cm

$$\text{Area of square} = (\text{side})^2 = \text{}^2 = \text{} \dots\dots \text{(I)}$$

Area of shaded region inside the square

$$= \text{Area of square ABCD} - \text{Area of sector C - BXD}$$

$$= \text{} - \frac{\theta}{360} \times \pi r^2$$

$$= \text{} - \frac{90}{360} \times \frac{3.14}{1} \times \frac{400}{1}$$

$$= \text{} - 314$$

$$= \text{}$$

$$\begin{aligned} \text{Radius of bigger sector} &= \text{Length of diagonal of square ABCD} \\ &= 20\sqrt{2} \end{aligned}$$

Area of the shaded regions outside the square

$$= \text{Area of sector A - PCQ} - \text{Area of square ABCD}$$

$$= A(\text{A - PCQ}) - A(\square \text{ABCD})$$

$$= \left(\frac{\theta}{360} \times \pi \times r^2 \right) - \text{}^2$$

$$= \frac{90}{360} \times 3.14 (20\sqrt{2})^2 - (20)^2$$

$$= \text{} - \text{}$$

$$= \text{}$$

\therefore total area of the shaded region = 86 + 228 = 314 sq.cm.

