

1 Rational and Irrational numbers



Let's recall.

We are familiar with Natural numbers, Whole numbers, Integers and Rational numbers.

Natural numbers

1, 2, 3, 4, ...

Whole numbers

0, 1, 2, 3, 4, ...

Integers

..., -4, -3, -2, -1, 0, 1, 2, 3, ...

Rational numbers

$\frac{-25}{3}$, $\frac{10}{-7}$, -4, 0, 3, 8, $\frac{32}{3}$, $\frac{67}{5}$, etc.

Rational numbers : The numbers of the form $\frac{m}{n}$ are called rational numbers.

Here, m and n are integers but n is not zero.

We have also seen that there are infinite rational numbers between any two rational numbers.

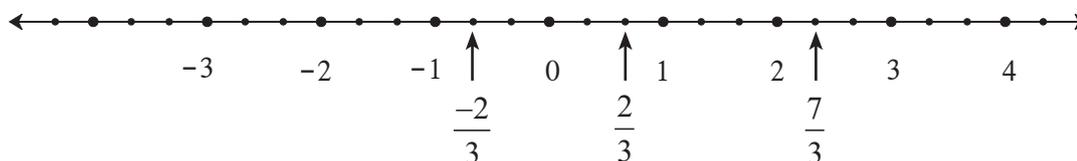


Let's learn.

To show rational numbers on a number line

Let us see how to show $\frac{7}{3}$, 2, $\frac{-2}{3}$ on a number line.

Let us draw a number line.



- We can show the number 2 on a number line.
- $\frac{7}{3} = 7 \times \frac{1}{3}$, therefore each unit on the right side of zero is to be divided in three equal parts. The seventh point from zero shows $\frac{7}{3}$; or $\frac{7}{3} = 2 + \frac{1}{3}$, hence the point at $\frac{1}{3}$ rd distance of unit after 2 shows $\frac{7}{3}$.

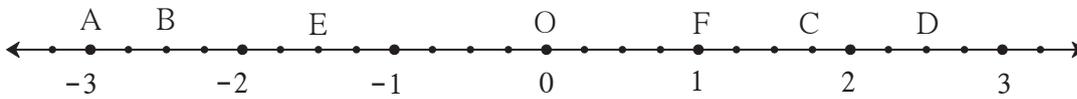
- To show $\frac{-2}{3}$ on the number line, first we show $\frac{2}{3}$ on it. The number to the left of 0 at the same distance will show the number $\frac{-2}{3}$.

Practice set 1.1

1. Show the following numbers on a number line. Draw a separate number line for each example.

(1) $\frac{3}{2}, \frac{5}{2}, -\frac{3}{2}$ (2) $\frac{7}{5}, \frac{-2}{5}, \frac{-4}{5}$ (3) $\frac{-5}{8}, \frac{11}{8}$ (4) $\frac{13}{10}, \frac{-17}{10}$

2. Observe the number line and answer the questions.



- Which number is indicated by point B?
- Which point indicates the number $1\frac{3}{4}$?
- State whether the statement, ‘the point D denotes the number $\frac{5}{2}$ ’, is true or false.



Comparison of rational numbers

We know that, for any pair of numbers on a number line the number to the left is smaller than the other. Also, if the numerator and the denominator of a rational number is multiplied by any non zero number then the value of rational number does not change. It remains the same. That is, $\frac{a}{b} = \frac{ka}{kb}$, ($k \neq 0$).

Ex. (1) Compare the numbers $\frac{5}{4}$ and $\frac{2}{3}$. Write using the proper symbol of $<$, $=$, $>$.

Solution : $\frac{5}{4} = \frac{5 \times 3}{4 \times 3} = \frac{15}{12}$ $\frac{2}{3} = \frac{2 \times 4}{3 \times 4} = \frac{8}{12}$

$\frac{15}{12} > \frac{8}{12}$ $\therefore \frac{5}{4} > \frac{2}{3}$

Ex. (2) Compare the rational numbers $\frac{-7}{9}$ and $\frac{4}{5}$.

Solution : A negative number is always less than a positive number.

$$\text{Therefore, } -\frac{7}{9} < \frac{4}{5}.$$

To compare two negative numbers,

let us verify that if a and b are positive numbers such that $a < b$, then $-a > -b$.

$$\left. \begin{array}{l} 2 < 3 \text{ but } -2 > -3 \\ \frac{5}{4} < \frac{7}{4} \text{ but } \frac{-5}{4} > \frac{-7}{4} \end{array} \right\} \text{Verify the comparisons using a number line.}$$

Ex. (3) Compare the numbers $\frac{-7}{3}$ and $\frac{-5}{2}$.

Solution : Let us first compare $\frac{7}{3}$ and $\frac{5}{2}$.

$$\frac{7}{3} = \frac{7 \times 2}{3 \times 2} = \frac{14}{6}, \quad \frac{5}{2} = \frac{5 \times 3}{2 \times 3} = \frac{15}{6} \quad \text{and} \quad \frac{14}{6} < \frac{15}{6}$$
$$\therefore \frac{7}{3} < \frac{5}{2} \quad \therefore \frac{-7}{3} > \frac{-5}{2}$$

Ex. (4) $\frac{3}{5}$ and $\frac{6}{10}$ are rational numbers. Compare them.

Solution : $\frac{3}{5} = \frac{3 \times 2}{5 \times 2} = \frac{6}{10} \quad \therefore \frac{3}{5} = \frac{6}{10}$

The following rules are useful to compare two rational numbers.

If $\frac{a}{b}$ and $\frac{c}{d}$ are rational numbers such that b and d are positive, and

(1) if $a \times d < b \times c$ then $\frac{a}{b} < \frac{c}{d}$

(2) if $a \times d = b \times c$ then $\frac{a}{b} = \frac{c}{d}$

(3) if $a \times d > b \times c$ then $\frac{a}{b} > \frac{c}{d}$

Practice Set 1.2

1. Compare the following numbers.

(1) $-7, -2$ (2) $0, \frac{-9}{5}$ (3) $\frac{8}{7}, 0$ (4) $\frac{-5}{4}, \frac{1}{4}$ (5) $\frac{40}{29}, \frac{141}{29}$

(6) $-\frac{17}{20}, \frac{-13}{20}$ (7) $\frac{15}{12}, \frac{7}{16}$ (8) $\frac{-25}{8}, \frac{-9}{4}$ (9) $\frac{12}{15}, \frac{3}{5}$ (10) $\frac{-7}{11}, \frac{-3}{4}$



Let's learn.

Decimal representation of rational numbers

If we use decimal fractions while dividing the numerator of a rational number by its denominator, we get the decimal representation of a rational number. For example, $\frac{7}{4} = 1.75$. In this case, after dividing 7 by 4, the remainder is zero. Hence the process of division ends.

Such a decimal form of a rational number is called a terminating decimal form.

We know that every rational number can be written in a non-terminating recurring decimal form.

For example, (1) $\frac{7}{6} = 1.1666\dots = 1.1\dot{6}$ (2) $\frac{5}{6} = 0.8333\dots = 0.8\dot{3}$

(3) $\frac{-5}{3} = -1.666\dots = -1.\dot{6}$

(4) $\frac{22}{7} = 3.142857142857\dots = 3.\overline{142857}$ (5) $\frac{23}{99} = 0.2323\dots = 0.\overline{23}$

Similarly, a terminating decimal form can be written as a non-terminating recurring decimal form. For example, $\frac{7}{4} = 1.75 = 1.75000\dots = 1.75\dot{0}$.

Practice Set 1.3

1. Write the following rational numbers in decimal form.

(1) $\frac{9}{37}$

(2) $\frac{18}{42}$

(3) $\frac{9}{14}$

(4) $\frac{-103}{5}$

(5) $-\frac{11}{13}$



Let's learn.

Irrational numbers

In addition to rational numbers, there are many more numbers on a number line. They are not rational numbers, that is, they are irrational numbers. $\sqrt{2}$ is such an irrational number.

We learn how to show the number $\sqrt{2}$ on a number line.

- On a number line, the point A shows the number 1. Draw line l perpendicular to the number line through point A.

Take point P on line l such that $OA = AP = 1$ unit.

- Draw seg OP. The Δ OAP formed is a right angled triangle.

By Pythagoras theorem,

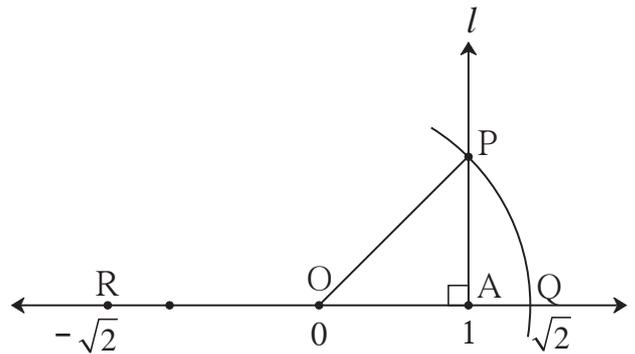
$$OP^2 = OA^2 + AP^2$$

$$= 1^2 + 1^2 = 1 + 1 = 2$$

$$OP^2 = 2$$

$\therefore OP = \sqrt{2}$...(taking square roots on both sides)

- Now, draw an arc with centre O and radius OP. Name the point as Q



where the arc intersects the number line. Obviously distance OQ is $\sqrt{2}$.

That is, the number shown by the point Q is $\sqrt{2}$.

If we mark point R on the number line to the left of O, at the same distance as OQ, then it will indicate the number $-\sqrt{2}$.

We will prove that $\sqrt{2}$ is an irrational number in the next standard. We will also see that the decimal form of an irrational number is non-terminating and non-recurring.

Note that -

In the previous standard we have learnt that π is not a rational number. It means it is irrational. For calculation purpose we take its value as $\frac{22}{7}$ or 3.14 which are very close to π ; but $\frac{22}{7}$ and 3.14 are rational numbers.

The numbers which can be shown by points of a number line are called real numbers. We have seen that all rational numbers can be shown by points of a number line. Therefore, all rational numbers are real numbers. There are infinitely many irrational numbers on the number line.

$\sqrt{2}$ is an irrational number. Note that the numbers like $3\sqrt{2}$, $7 + \sqrt{2}$, $3 - \sqrt{2}$ etc. are also irrational numbers; because if $3\sqrt{2}$ is rational then $\frac{3\sqrt{2}}{3}$ should also be a rational number, which is not true.

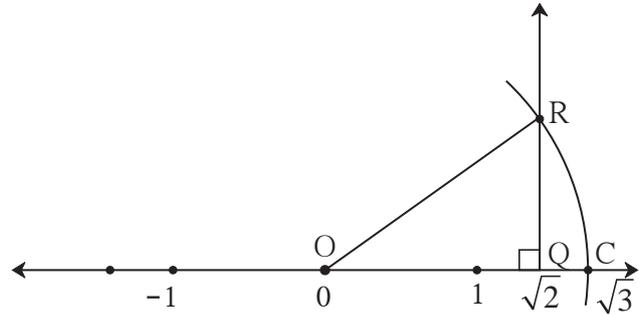
We learnt to show rational numbers on a number line. We have shown the irrational number $\sqrt{2}$ on a number line. Similarly we can show irrational numbers like $\sqrt{3}$, $\sqrt{5}$. . . on a number line.

Practice Set 1.4

1. The number $\sqrt{2}$ is shown on a number line. Steps are given to show $\sqrt{3}$ on the number line using $\sqrt{2}$. Fill in the boxes properly and complete the activity.

Activity :

- The point Q on the number line shows the number
- A line perpendicular to the number line is drawn through the point Q. Point R is at unit distance from Q on the line.
- Right angled Δ ORQ is obtained by drawing seg OR.
- $l(OQ) = \sqrt{2}$, $l(QR) = 1$



\therefore by Pythagoras theorem,

$$\begin{aligned}
 [l(OR)]^2 &= [l(OQ)]^2 + [l(QR)]^2 \\
 &= \boxed{}^2 + \boxed{}^2 = \boxed{} + \boxed{} \\
 &= \boxed{} \quad \therefore l(OR) = \boxed{}
 \end{aligned}$$

Draw an arc with centre O and radius OR. Mark the point of intersection of the line and the arc as C. The point C shows the number $\sqrt{3}$.

2. Show the number $\sqrt{5}$ on the number line.
- 3* Show the number $\sqrt{7}$ on the number line.



Answers

Practice Set 1.1

2. (1) $\frac{-10}{4}$ (2) C (3) True

Practice Set 1.2

1. (1) $-7 < -2$ (2) $0 > \frac{-9}{5}$ (3) $\frac{8}{7} > 0$ (4) $\frac{-5}{4} < \frac{1}{4}$ (5) $\frac{40}{29} < \frac{141}{29}$
- (6) $\frac{-17}{20} < \frac{-13}{20}$ (7) $\frac{15}{12} > \frac{7}{16}$ (8) $\frac{-25}{8} < \frac{-9}{4}$ (9) $\frac{12}{15} > \frac{3}{5}$
- (10) $\frac{-7}{11} > \frac{-3}{4}$

Practice Set 1.3

- (1) $0.\overline{243}$ (2) $0.\overline{428571}$ (3) $0.\overline{6428571}$ (4) -20.6 (5) $-0.\overline{846153}$

