

Activity : Verify the following examples.

- (1) Divide $p(x) = 3x^2 + x + 7$ by $x + 2$. Find the Remainder.
- (2) Find the value of $p(x) = 3x^2 + x + 7$ when $x = -2$.
- (3) See whether remainder obtained by division is same as the value of $p(-2)$. Take one more example and verify.

Ex (1) Divide $x^4 - 5x^2 - 4x$ by $x + 3$ and find the remainder.

Solution : By Remainder Theorem

Dividend polynomial $p(x) = x^4 - 5x^2 - 4x$

Divisor = $x + 3$

take $x = -3$.

$$\therefore p(x) = x^4 - 5x^2 - 4x$$

$$p(-3) = (-3)^4 - 5(-3)^2 - 4(-3)$$

$$= 81 - 45 + 12$$

$$p(-3) = 48$$

By Synthetic Divison

Index form $x^4 + 0x^3 - 5x^2 - 4x + 0$

Coefficient form = $(1, 0, -5, -4, 0)$

- 3	1	0	-5	-4	0
		-3	9	-12	48
	1	-3	4	-16	48

Remainder = 48

Ex (2) By using remainder theorem divide the polynomial $x^3 - 2x^2 - 4x - 1$ by $x - 1$ and find the remainder.

Solution : $p(x) = x^3 - 2x^2 - 4x - 1$

Divisor = $x - 1$ \therefore take $x = 1$

$$\therefore \text{Remainder} = p(1) = 1^3 - 2 \times 1^2 - 4 \times 1 - 1 \quad \dots(\text{by remainder theorem})$$

$$= 1 - 2 \times 1 - 4 - 1$$

$$p(1) = 1 - 2 - 4 - 1 = -6$$

$$\therefore \text{Remainder} = -6$$

Ex (3) If the polynomial $t^3 - 3t^2 + kt + 50$ is divided by $(t-3)$, the remainder is 62. Find the value of k .

Solution : When given polynomial is divided by $(t-3)$ the remainder is 62. It means the value of the polynomial when $t = 3$ is 62.

$$p(t) = t^3 - 3t^2 + kt + 50$$

By remainder theorem,

$$\begin{aligned}\text{Remainder} &= p(3) = 3^3 - 3 \times 3^2 + k \times 3 + 50 \\ &= 27 - 3 \times 9 + 3k + 50 \\ &= 27 - 27 + 3k + 50 \\ &= 3k + 50\end{aligned}$$

But remainder is 62.

$$\therefore 3k + 50 = 62$$

$$\therefore 3k = 62 - 50$$

$$\therefore 3k = 12$$

$$\therefore k = \frac{12}{3}$$

$$\therefore k = 4$$



Remember this !

If a polynomial $p(x)$ is divided by $(x + a)$ then the remainder is $p(-a)$ where 'a' is a real number.

$$p(x) = s(x) \times (x - a) + r(x) \text{ where degree of } r(x) < 1 \text{ or } r(x) = 0$$

In this equation by putting $x = a$ we get, $p(a) = 0 + r(a) = r(a)$.

Hence if $r(a) = 0$ means $(x - a)$ is a factor of $p(x)$.



Let's learn.

Factor Theorem

If 21 is divided by 7 then remainder is 0, therefore we say that 7 is a factor of 21. In the same way when a given polynomial is divided by the divisor polynomial and if the remainder is 0 then we say that divisor polynomial is the factor of the dividend polynomial.

Ex (1) If $p(x) = (x^3 + 4x - 5)$ is divided by $(x - 1)$ then find the remainder and hence determine whether $(x - 1)$ is a factor of $p(x)$ or not ?

$$\begin{aligned}\text{Solution : } p(x) &= x^3 + 4x - 5 \\ p(1) &= (1)^3 + 4(1) - 5 \\ &= 1 + 4 - 5 \\ &= 0\end{aligned}$$

As per the remainder theorem,

$$\text{Remainder} = 0$$

$\therefore (x - 1)$ is a factor of $p(x)$.

Ex (2) If $p(x) = x^3 + 4x - 5$ is divided by $x + 2$ then find the remainder and hence determine whether $(x + 2)$ is a factor of $p(x)$ or not.

$$\begin{aligned}\text{Solution : } p(x) &= x^3 + 4x - 5 \\ p(-2) &= (-2)^3 + 4(-2) - 5 \\ p(-2) &= -8 - 8 - 5 \\ &= -21\end{aligned}$$

As per the remainder theorem,

$$\text{Remainder} = -21 \therefore \text{Remainder} \neq 0$$

$\therefore (x + 2)$ is not a factor of $p(x)$.

Activity : Verify that $(x - 1)$ is a factor of the polynomial $x^3 + 4x - 5$.



Remember this !

$p(x)$ is a polynomial and a is any real number, and if $p(a) = 0$ then $(x - a)$ is the factor of $p(x)$.

Conversely if $(x - a)$ is the factor of the polynomial $p(x)$ then $p(a) = 0$.

Ex (1) Check whether, $x - 2$ is a factor of the polynomial $x^3 - x^2 - 4$ by using factor theorem.

Solution : $p(x) = x^3 - x^2 - 4$ Divisor = $x - 2$

$$\therefore p(2) = 2^3 - 2^2 - 4 = 8 - 4 - 4 = 0$$

\therefore By factor theorem $(x - 2)$ is a factor of the polynomial $(x^3 - x^2 - 4)$.

Ex (2) If $(x - 1)$ is the factor of the polynomial $(x^3 - 2x^2 + mx - 4)$ then find the value of m .

Solution : $(x - 1)$ is factor of $p(x)$. $\therefore p(1) = 0$

$$p(x) = x^3 - 2x^2 + mx - 4$$

$$p(1) = 1^3 - 2 \times 1^2 + m \times 1 - 4 = 0$$

$$\therefore 1 - 2 \times 1 + m - 4 = 0$$

$$\therefore 1 - 2 + m - 4 = 0 \quad \therefore m - 5 = 0 \quad \therefore m = 5$$

Activity : We have seen the example of expenditure and income (in terms of polynomials) of Govind who is a dry land farmer. He has borrowed rupees one lakh twenty, five thousand from the bank as an agriculture loan and repaid the said loan at 10 p.c.p.a. He had spent ₹ 10,000 on seeds. The expenses on soyabean crop was ₹ 2000x for fertilizers and pesticides and ₹ 4000x² was spent on wages and cultivation. He spent ₹ 8000y on fertilizers and pesticides and ₹ 9000y² on cultivation and wages for cotton and tur crop.

His total income was rupees $14000x^2 + \frac{25000}{3}y^2 + 16000y$.

By taking $x = 2, y = 3$ write the income-expenditure account of Govind's farming.

Solution : **Credit (Income)**

Debit (Expenses)

₹ 1,25,000	Bank loan	₹ 1,37,000	loan paid with interest for seeds
₹ <input type="text"/>	Income from soyabean	₹ <input type="text"/>	For seeds
₹ <input type="text"/>	Income from cotton	₹ <input type="text"/>	Fertilizers and pesticides for soyabean
₹ <input type="text"/>	Income from tur	₹ <input type="text"/>	Wages and cultivation charges for soyabean
₹ <input type="text"/>	Total income	₹ <input type="text"/>	Fertilizers and pesticides for cotton & tur
		₹ <input type="text"/>	Wages and cultivation charges for cotton & tur
		₹ <input type="text"/>	Total expenditure

Practice set 3.5

- (1) Find the value of the polynomial $2x - 2x^3 + 7$ using given values for x .
- (i) $x = 3$ (ii) $x = -1$ (iii) $x = 0$
- (2) For each of the following polynomial, find $p(1)$, $p(0)$ and $p(-2)$.
- (i) $p(x) = x^3$ (ii) $p(y) = y^2 - 2y + 5$ (iii) $p(x) = x^4 - 2x^2 - x$
- (3) If the value of the polynomial $m^3 + 2m + a$ is 12 for $m = 2$, then find the value of a .
- (4) For the polynomial $mx^2 - 2x + 3$ if $p(-1) = 7$ then find m .
- (5) Divide the first polynomial by the second polynomial and find the remainder using remainder theorem.
- (i) $(x^2 - 7x + 9) ; (x + 1)$ (ii) $(2x^3 - 2x^2 + ax - a) ; (x - a)$
 (iii) $(54m^3 + 18m^2 - 27m + 5) ; (m - 3)$
- (6) If the polynomial $y^3 - 5y^2 + 7y + m$ is divided by $y + 2$ and the remainder is 50 then find the value of m .
- (7) Use factor theorem to determine whether $x + 3$ is factor of $x^2 + 2x - 3$ or not.
- (8) If $(x - 2)$ is a factor of $x^3 - mx^2 + 10x - 20$ then find the value of m .
- (9) By using factor theorem in the following examples, determine whether $q(x)$ is a factor $p(x)$ or not.
- (i) $p(x) = x^3 - x^2 - x - 1$, $q(x) = x - 1$
 (ii) $p(x) = 2x^3 - x^2 - 45$, $q(x) = x - 3$
- (10) If $(x^{31} + 31)$ is divided by $(x + 1)$ then find the remainder.
- (11) Show that $m - 1$ is a factor of $m^{21} - 1$ and $m^{22} - 1$.
- (12*) If $x - 2$ and $x - \frac{1}{2}$ both are the factors of the polynomial $nx^2 - 5x + m$, then show that $m = n = 2$
- (13) (i) If $p(x) = 2 + 5x$ then $p(2) + p(-2) - p(1)$.
 (ii) If $p(x) = 2x^2 - 5\sqrt{3}x + 5$ then $p(5\sqrt{3})$.



Let's recall.

In previous classes we have learnt how to find the factors of the polynomials. Let's revise it with some examples.

Factorize.

Ex (1) $4x^2 - 25$
 $= (2x)^2 - (5)^2$
 $= (2x + 5)(2x - 5)$

Ex (2) $3x^2 + 7x + 2$
 $= \frac{3x^2 + 6x + x + 2}{= 3x(x + 2) + 1(x + 2)}$
 $= (x + 2)(3x + 1)$

$$\begin{aligned}
 \text{Ex (3)} \quad & 63x^2 + 5x - 2 \\
 &= 63x^2 + 14x - 9x - 2 \\
 &= 7x(9x + 2) - 1(9x + 2) \\
 &= (9x + 2)(7x - 1)
 \end{aligned}$$

$$\begin{aligned}
 \text{Ex (4)} \quad & 6x^2 - 5x - 6 \\
 &= 6x^2 - 9x + 4x - 6 \\
 &= 3x(2x - 3) + 2(2x - 3) \\
 &= (2x - 3)(3x + 2)
 \end{aligned}$$



Let's learn.

Factors of polynomials

Sometimes polynomial can be written in the form $ax^2 + bx + c$ and hence it is easy to find its factors.

Ex (1) Factorise : $(y^2-3y)^2 - 5(y^2-3y) - 50$.

Solution : Let $(y^2-3y) = x$

$$\begin{aligned}
 \therefore (y^2-3y)^2 - 5(y^2-3y) - 50 &= x^2 - 5x - 50 \\
 &= x^2 - 10x + 5x - 50 \\
 &= x(x - 10) + 5(x - 10) \\
 &= (x - 10)(x + 5) \\
 &= (y^2-3y - 10)(y^2-3y + 5) \\
 &= [y^2-5y + 2y - 10](y^2-3y + 5) \\
 &= [y(y - 5) + 2(y - 5)](y^2-3y + 5) \\
 &= (y - 5)(y + 2)(y^2-3y + 5)
 \end{aligned}$$

Ex (2) Factorise.

$$(x + 2)(x - 3)(x - 7)(x - 2) + 64$$

$$\begin{aligned}
 \text{Solution : } & (x + 2)(x - 3)(x - 7)(x - 2) + 64 \\
 &= (x + 2)(x - 7)(x - 3)(x - 2) + 64 \\
 &= (x^2 - 5x - 14)(x^2 - 5x + 6) + 64 \\
 &= (m - 14)(m + 6) + 64 \dots \dots \dots \text{ (putting } x^2 - 5x = m) \\
 &= m^2 - 14m + 6m - 84 + 64 \\
 &= m^2 - 8m - 20 \\
 &= (m - 10)(m + 2) \\
 &= (x^2 - 5x - 10)(x^2 - 5x + 2) \dots \text{ (replace } m \text{ with } x^2 - 5x)
 \end{aligned}$$

Practice set 3.6

(1) Find the factors of the polynomials given below.

(i) $2x^2 + x - 1$

(ii) $2m^2 + 5m - 3$

(iii) $12x^2 + 61x + 77$

(iv) $3y^2 - 2y - 1$

(v) $\sqrt{3}x^2 + 4x + \sqrt{3}$

(vi) $\frac{1}{2}x^2 - 3x + 4$

(2) Factorize the following polynomials.

(i) $(x^2 - x)^2 - 8(x^2 - x) + 12$

(ii) $(x - 5)^2 - (5x - 25) - 24$

(iii) $(x^2 - 6x)^2 - 8(x^2 - 6x + 8) - 64$

(iv) $(x^2 - 2x + 3)(x^2 - 2x + 5) - 35$

(v) $(y + 2)(y - 3)(y + 8)(y + 3) + 56$

(vi) $(y^2 + 5y)(y^2 + 5y - 2) - 24$

(vii) $(x - 3)(x - 4)^2(x - 5) - 6$

Problem set 3

(1) Write the correct alternative answer for each of the following questions.

(i) Which of the following is a polynomial ?

- (A) $\frac{x}{y}$ (B) $x^{\sqrt{2}} - 3x$ (C) $x^{-2} + 7$ (D) $\sqrt{2}x^2 + \frac{1}{2}$

(ii) What is the degree of the polynomial $\sqrt{7}$?

- (A) $\frac{1}{2}$ (B) 5 (C) 2 (D) 0

(iii) What is the degree of the 0 polynomial ?

- (A) 0 (B) 1 (C) undefined (D) any real number

(iv) What is the degree of the polynomial $2x^2 + 5x^3 + 7$?

- (A) 3 (B) 2 (C) 5 (D) 7

(v) What is the coefficient form of $x^3 - 1$?

- (A) (1, - 1) (B) (3, - 1) (C) (1, 0, 0, - 1) (D) (1, 3, - 1)

(vi) $p(x) = x^2 - 7\sqrt{7}x + 3$ then $p(7\sqrt{7}) = ?$

- (A) 3 (B) $7\sqrt{7}$ (C) $42\sqrt{7} + 3$ (D) $49\sqrt{7}$

(vii) When $x = - 1$, what is the value of the polynomial $2x^3 + 2x$?

- (A) 4 (B) 2 (C) - 2 (D) - 4

(viii) If $x - 1$ is a factor of the polynomial $3x^2 + mx$ then find the value of m .

- (A) 2 (B) - 2 (C) - 3 (D) 3

(ix) Multiply $(x^2 - 3)(2x - 7x^3 + 4)$ and write the degree of the product.

- (A) 5 (B) 3 (C) 2 (D) 0

- (x) Which of the following is a linear polynomial ?
 (A) $x + 5$ (B) $x^2 + 5$ (C) $x^3 + 5$ (D) $x^4 + 5$
- (2) Write the degree of the polynomial for each of the following.
 (i) $5 + 3x^4$ (ii) 7 (iii) $ax^7 + bx^9$ (a, b are constants.)
- (3) Write the following polynomials in standard form.
 (i) $4x^2 + 7x^4 - x^3 - x + 9$ (ii) $p + 2p^3 + 10p^2 + 5p^4 - 8$
- (4) Write the following polynomial in coefficient form.
 (i) $x^4 + 16$ (ii) $m^5 + 2m^2 + 3m + 15$
- (5) Write the index form of the polynomial using variable x from its coefficient form.
 (i) $(3, -2, 0, 7, 18)$ (ii) $(6, 1, 0, 7)$ (iii) $(4, 5, -3, 0)$
- (6) Add the following polynomials.
 (i) $7x^4 - 2x^3 + x + 10$; $3x^4 + 15x^3 + 9x^2 - 8x + 2$
 (ii) $3p^3q + 2p^2q + 7$; $2p^2q + 4pq - 2p^3q$
- (7) Subtract the second polynomial from the first.
 (i) $5x^2 - 2y + 9$; $3x^2 + 5y - 7$ (ii) $2x^2 + 3x + 5$; $x^2 - 2x + 3$
- (8) Multiply the following polynomials.
 (i) $(m^3 - 2m + 3)(m^4 - 2m^2 + 3m + 2)$ (ii) $(5m^3 - 2)(m^2 - m + 3)$
- (9) Divide polynomial $3x^3 - 8x^2 + x + 7$ by $x - 3$ using synthetic method and write the quotient and remainder.
- (10) For which the value of m , $x + 3$ is the factor of the polynomial $x^3 - 2mx + 21$?
- (11) At the end of the year 2016, the population of villages Kovad, Varud, Chikhali is $5x^2 - 3y^2$, $7y^2 + 2xy$ and $9x^2 + 4xy$ respectively. At the beginning of the year 2017, $x^2 + xy - y^2$, $5xy$ and $3x^2 + xy$ persons from each of the three villages respectively went to another village for education then what is the remaining total population of these three villages ?
- (12) Polynomials $bx^2 + x + 5$ and $bx^3 - 2x + 5$ are divided by polynomial $x - 3$ and the remainders are m and n respectively. If $m - n = 0$ then find the value of b .
- (13) Simplify. $(8m^2 + 3m - 6) - (9m - 7) + (3m^2 - 2m + 4)$
- (14) Which polynomial is to be subtracted from $x^2 + 13x + 7$ to get the polynomial $3x^2 + 5x - 4$?
- (15) Which polynomial is to be added to $4m + 2n + 3$ to get the polynomial $6m + 3n + 10$?



4

Ratio and Proportion



Let's study.

- Ratio
- Operations on equal ratios
- Continued proportion
- Properties of ratios
- Theorem of equal ratios
- k method



Let's recall.

In earlier standards, we have learnt about ratio and proportion. We have also solved examples based on it. Let us discuss following example.

Ex. The rawa laddoo prepared by Vimal are tasty, for which she takes 1 bowl of ghee, 3 bowls of rawa and 2 bowls of sugar.

Here proportion of rawa and sugar is $3 : 2$ or $\frac{3}{2}$.

If 12 units of rawa is used, how many units of sugar are required ?

Let the number of bowls of sugar required be x .

\therefore from above information, $\frac{3}{2} = \frac{12}{x} \quad \therefore 3x = 24 \quad \therefore x = 8$

That is for preparation of laddoo, with 12 units of rawa requires 8 units of sugar. Alternatively we can solve the above example in the following way.

$3k$ bowls of rawa, $2k$ bowls of sugar is required because $\frac{3k}{2k} = \frac{3}{2}$

If $3k = 12$ then $k = 4 \quad \therefore 2k = 2 \times 4 = 8$ bowls of sugar is required.



Let's learn.

Ratio and proportion

The concept of ratio of two numbers can be extended to three or more numbers.

Let us see the above example of laddoos. The proportion of ghee, rawa and sugar is $1 : 3 : 2$.

Here proportion of ghee and rawa is $1 : 3$ and that of rawa and sugar is $3 : 2$, This means the proportion of ghee, rawa and sugar is $1 : 3 : 2$.

Let us take k bowls of ghee, $3k$ bowls of rawa and $2k$ bowls of sugar.

Hence for 12 bowls of rawa, how much quantity of ghee and sugar is required can be found as follows.

Now $3k = 12 \quad \therefore k = 4$ and $2k = 8$.

\therefore 4 bowls of ghee and 8 bowls of sugar is required.

The same concept can be extended for proportion of 4 or more entities.

If a, b, c, d are in the ratio $2 : 3 : 7 : 4$ then let us assume that the numbers are $2m, 3m, 7m, 4m$. From the given information, value of m can be determined. For example if the sum of these four numbers is 48, we find these numbers.

$$2m + 3m + 7m + 4m = 16m = 48$$

$$\therefore m = 3$$

$$\therefore 2m = 6, 3m = 9, 7m = 21, 4m = 12$$

\therefore required numbers are 6, 9, 21, 12

Ex (1) The proportion of compounds of nitrogen, phosphorous and potassium in certain fertilizer is $18 : 18 : 10$. Here compound of nitrogen is 18%, compound of phosphorous is 18% and that of potassium is 10%. Remaining part is of other substances. Find the weight of each of the above compounds in 20 kg of fertilizer.

Solution : Let the weight of nitrogen compound in 20 kg of fertilizer be x kg.

$$\therefore \frac{18}{100} = \frac{x}{20} \qquad \therefore x = \frac{18 \times 20}{100} = 3.6$$

\therefore weight of nitrogen compound is 3.6 kg.

The percentage of phosphorous compound is also 18%.

\therefore Weight of compound of phosphorous is 3.6 kg.

If we assume the weight of potassium compound y kg then

$$\frac{10}{100} = \frac{y}{20} \quad \therefore y = 2 \qquad \therefore \text{weight of potassium compound is 2 kg.}$$

Direct proportion

A car covers a distance of 10 km consuming 1 litre of petrol.

It will cover a distance of $20 \times 10 = 200$ km consuming 20 litre of petrol .

Consuming 40 litre of petrol, it will cover a distance of $40 \times 10 = 400$ km.

Let us write this information in tabular form.

Petrol : x litre	1	20	40	
Distance : y km	10	200	400	
$\frac{x}{y}$	$\frac{1}{10}$	$\frac{20}{200} = \frac{1}{10}$	$\frac{40}{400} = \frac{1}{10}$	$\frac{x}{y} = k$

The ratio of consumption of petrol (in litre) and distance covered by the car (in kilometres), is constant. In such case, it is said that the two quantities are in direct proportion or in direct variation.

Inverse proportion

A car takes two hours to cover a distance of 100 km at the speed of 50 km/hr. A bullock-cart travels 5 km in 1 hour. To cover a distance of 100 km at the speed of 5 km/hr, the bullock-cart takes 20 hours.

We know that, **Speed \times time = distance**

By using the relation let us put the above information in a tabular form.

Vehicle	Speed/hr (x)	Time (y)	$x \times y$	$x \times y = k$
Car	50	2	100	
Bullock-cart	5	20	100	

Hence, we see that, the product of speed of the vehicle and time is constant. In such a case it is said that the quantities are in inverse proportion or in inverse variation.



Let's recall.

Properties of ratio

- (1) Ratio of numbers a and b is written as $a : b$ or $\frac{a}{b}$. a is called the predecessor (first term) and b is called successor (Second term).
- (2) In the ratio of two numbers, if the second term is 100 then it is known as a percentage.
- (3) The ratio remains unchanged, if its terms are multiplied or divided by non-zero number. e.g.. $3 : 4 = 6:8 = 9:12$, Similarly $2:3:5 = 8:12:20$. If k is a non-zero number, then $a : b = ak : bk$ $a : b : c = ak : bk : ck$
- (4) The quantities taken in the ratio must be expressed in the same unit.
- (5) The ratio of two quantities is unitless.

For example The ratio of 2 kg and 300 g is not $2 : 300$,
but it is $2000 : 300$ as ($2 \text{ kg} = 2000 \text{ gm}$) i.e. $20 : 3$

Ex (1) The ratio of ages of Seema and Rajashree is $3 : 1$. The ratio of ages of Rajashree and Atul is $2 : 3$. Then find the ratio of ages of Seema, Rajashree and Atul.

Solution : Seema's age : Rajashree's age = $3 : 1$ Rajashree's age : Atul's age = $2 : 3$

Second term of first ratio should be the same as the first term of second ratio.

Hence to get the continuous ratio, multiplying each term of the first ratio by 2. We get

$$3:1 = 6:2 .$$

$$\frac{\text{Seema's age}}{\text{Rajashree's age}} = \frac{6}{2}, \quad \frac{\text{Rajashree's age}}{\text{Atul's age}} = \frac{2}{3}$$

$$\therefore \text{Seema's age} : \text{Rajashree's age} : \text{Atul's age} = 6 : 2 : 3.$$

Ex (2) The length of a rectangular field is 1.2 km and its breadth is 400 metre. Find the ratio of length to breadth.

Solution : Here the length is in kilometer and breadth is in meter. In order to find the ratio of length to breadth, they must be expressed in same unit. Hence we convert kilometre to meter.

$$1.2 \text{ km} = 1.2 \times 1000 = 1200 \text{ m}$$

$$\therefore \text{ratio of } 1200 \text{ m, to } 400 \text{ m is } \frac{1200}{400} = \frac{3}{1}, \text{ that is } 3 : 1$$

Ex (3) The ratio of expenditure and income of Mahesh is 3 : 5. Find the percentage of expenses to his income.

Solution : The ratio of expenditure to income is 3 : 5. To convert it into percentage, convert second term into 100.

$$\frac{3}{5} = \frac{3 \times 20}{5 \times 20} = \frac{60}{100} \therefore \frac{\text{Expenditure}}{\text{Income}} = \frac{60}{100} = 60\% \therefore \text{Mahesh spends } 60\% \text{ of his income.}$$

Ex (4) The ratio of number of mango trees to chikoo trees in an orchard is 2 : 3. If 5 more trees of each type are planted the ratio of trees would be 5 : 7. Then find the number of mango and chickoo trees in the orchard.

Solution : The ratio of trees is 2 : 3.

Let the number of mango trees = $2x$ and chikoo trees = $3x$

$$\text{From given condition, } \frac{2x+5}{3x+5} = \frac{5}{7}$$

$$14x + 35 = 15x + 25$$

$$\therefore x = 10$$

$$\therefore \text{number of mango trees in the orchard} = 2x = 2 \times 10 = 20$$

$$\text{and number of chikoo trees} = 3x = 3 \times 10 = 30$$

Ex (5) The ratio of two numbers is 5 : 7. If 40 is added in each number, then the ratio becomes 25 : 31, Find the numbers.

Solution : Let the first number be $5x$ and second number be $7x$.

From the given condition, $\frac{5x+40}{7x+40} = \frac{25}{31}$

$$31(5x+40) = 25(7x+40)$$

$$155x+1240 = 175x+1000$$

$$1240-1000 = 175x-155x$$

$$240 = 20x$$

$$x = 12$$

\therefore first number = $5 \times 12 = 60$ and second number = $7 \times 12 = 84$

\therefore given numbers are 60 and 84.

Practice set 4.1

- (1) From the following pairs of numbers, find the reduced form of ratio of first number to second number.
(i) 72, 60 (ii) 38,57 (iii) 52,78
- (2) Find the reduced form of the ratio of the first quantity to second quantity.
(i) 700 ₹, 308 ₹ (ii) 14 ₹, 12 ₹. 40 paise.
(iii) 5 litre, 2500 ml (iv) 3 years 4 months, 5 years 8 months
(v) 3.8 kg, 1900 gm (vi) 7 minutes 20 seconds, 5 minutes 6 seconds.
- (3) Express the following percentages as ratios in the reduced form.
(i) 75 : 100 (ii) 44 : 100 (iii) 6.25% (iv) 52 : 100 (v) 0.64%
- (4) Three persons can build a small house in 8 days. To build the same house in 6 days, how many persons are required?
- (5) Convert the following ratios into percentage.
(i) 15 : 25 (ii) 47 : 50 (iii) $\frac{7}{10}$ (iv) $\frac{546}{600}$ (v) $\frac{7}{16}$
- (6) The ratio of ages of Abha and her mother is 2 : 5. At the time of Abha's birth her mother's age was 27 year. Find the present ages of Abha and her mother.
- (7) Present ages of Vatsala and Sara are 14 years and 10 years respectively. After how many years the ratio of their ages will become 5 : 4?
- (8) The ratio of present ages of Rehana and her mother is 2 : 7. After 2 years, the ratio of their ages will be 1 : 3. What is Rehana's present age ?



Let's learn.

Comparison of ratios

The numbers a, b, c, d being positive, comparison of ratios $\frac{a}{b}, \frac{c}{d}$ can be done using following rules :

(i) If $ad > bc$ then $\frac{a}{b} > \frac{c}{d}$ (ii) If $ad < bc$ then $\frac{a}{b} < \frac{c}{d}$

(iii) If $ad = bc$ then $\frac{a}{b} = \frac{c}{d}$

Compare the following pairs of ratios.

Ex (1) $\frac{4}{9}, \frac{7}{8}$

Solution : $4 \times 8 \quad [?] \quad 7 \times 9$
 $32 < 63$
 $\therefore \frac{4}{9} < \frac{7}{8}$

Ex (2) $\frac{\sqrt{13}}{\sqrt{8}}, \frac{\sqrt{7}}{\sqrt{5}}$
 $\sqrt{13} \times \sqrt{5}, \quad [?] \quad \sqrt{8} \times \sqrt{7}$

$\sqrt{65} \quad [?] \quad \sqrt{56}$

$\sqrt{65} > \sqrt{56}$

$\therefore \frac{\sqrt{13}}{\sqrt{8}} > \frac{\sqrt{7}}{\sqrt{5}}$

Ex (3) If a and b are integers and $a < b, b > 1$ then compare $\frac{a-1}{b-1}, \frac{a+1}{b+1}$.

Solution : $a < b \quad \therefore a - 1 < b - 1$

Now consider the subtraction $\frac{a-1}{b-1} - \frac{a+1}{b+1}$

$$\begin{aligned} \frac{a-1}{b-1} - \frac{a+1}{b+1} &= \frac{(a-1)(b+1) - (a+1)(b-1)}{(b-1)(b+1)} \\ &= \frac{(ab-b+a-1) - (ab+b-a-1)}{b^2-1} \\ &= \frac{ab-b+a-1-ab-b+a+1}{b^2-1} \\ &= \frac{2a-2b}{b^2-1} \\ &= \frac{2(a-b)}{b^2-1} \dots\dots\dots (1) \end{aligned}$$

Now $a < b \quad \therefore a - b < 0$

also $b^2-1 > 0$ because $b > 1$

$\frac{2(a-b)}{b^2-1} < 0 \dots\dots\dots (2)$

$\frac{a-1}{b-1} - \frac{a+1}{b+1} < 0 \dots\dots$ from (1) & (2)

$\frac{a-1}{b-1} < \frac{a+1}{b+1}$

Ex (4) If $a : b = 2 : 1$ and $b : c = 4 : 1$ then find the value of $\left(\frac{a^4}{32b^2c^2}\right)^3$.

Solution : $\frac{a}{b} = \frac{2}{1} \quad \therefore a = 2b \quad \frac{b}{c} = \frac{4}{1} \quad \therefore b = 4c$

$$a = 2b = 2 \times 4c = 8c \quad \therefore a = 8c$$

Now substituting the values $a = 8c$, $b = 4c$

$$\left(\frac{a^4}{32b^2c^2}\right)^3 = \left(\frac{(8c)^4}{32 \times 4^2 \times c^2 \times c^2}\right)^3$$

$$= \left[\frac{8 \times 8 \times 8 \times 8 \times c^4}{32 \times 16 \times c^2 \times c^2}\right]^3$$

$$= (8)^3$$

$$\therefore \left(\frac{a^4}{32b^2c^2}\right)^3 = 512$$

Practice set 4.2

(1) Using the property $\frac{a}{b} = \frac{ak}{bk}$, fill in the blanks substituting proper numbers in the following.

(i) $\frac{5}{7} = \frac{\dots}{28} = \frac{35}{\dots} = \frac{\dots}{3.5}$

(ii) $\frac{9}{14} = \frac{4.5}{\dots} = \frac{\dots}{42} = \frac{\dots}{3.5}$

(2) Find the following ratios.

(i) The ratio of radius to circumference of the circle.

(ii) The ratio of circumference of circle with radius r to its area.

(iii) The ratio of diagonal of a square to its side, if the length of side is 7 cm.

(iv) The lengths of sides of a rectangle are 5 cm and 3.5 cm. Find the ratio of its perimeter to area.

(3) Compare the following pairs of ratios.

(i) $\frac{\sqrt{5}}{3}, \frac{3}{\sqrt{7}}$

(ii) $\frac{3\sqrt{5}}{5\sqrt{7}}, \frac{\sqrt{63}}{\sqrt{125}}$

(iii) $\frac{5}{18}, \frac{17}{121}$

(iv) $\frac{\sqrt{80}}{\sqrt{48}}, \frac{\sqrt{45}}{\sqrt{27}}$

(v) $\frac{9.2}{5.1}, \frac{3.4}{7.1}$

(4) (i) $\square ABCD$ is a parallelogram. The ratio of $\angle A$ and $\angle B$ of this parallelogram is 5 : 4. Find the measure of $\angle B$.

(ii) The ratio of present ages of Albert and Salim is 5 : 9. Five years hence ratio of their ages will be 3 : 5. Find their present ages.

(iii) The ratio of length and breadth of a rectangle is 3 : 1, and its perimeter is 36 cm. Find the length and breadth of the rectangle.

(iv) The ratio of two numbers is 31 : 23 and their sum is 216. Find these numbers.

(v) If the product of two numbers is 360 and their ratio is 10 : 9, then find the numbers.

(5*) If $a : b = 3 : 1$ and $b : c = 5 : 1$ then find the value of (i) $\left(\frac{a^3}{15b^2c}\right)^3$ (ii) $\frac{a^2}{7bc}$

(6*) If $\sqrt{0.04 \times 0.4 \times a} = 0.4 \times 0.04 \times \sqrt{b}$ then find the ratio $\frac{a}{b}$.

(7) $(x + 3) : (x + 11) = (x - 2) : (x + 1)$ then find the value of x .



Let's learn.

Operations on equal ratios

Using the properties of equality, we can perform some operations on ratios. Let's study them.

Let us learn some properties of the equal ratios, if a, b, c, d , are positive integers.

(I) Invertendo : If $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$

$$\begin{aligned} \frac{a}{b} &= \frac{c}{d} \\ \therefore a \times d &= b \times c \\ \therefore b \times c &= a \times d \\ \therefore \frac{b \times c}{a \times c} &= \frac{a \times d}{a \times c} \quad \dots(\text{dividing both sides by } a \times c) \\ \frac{b}{a} &= \frac{d}{c} \end{aligned}$$

\therefore If $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$. **This property is known as Invertendo.**

(II) Alternando : If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$

$$\begin{aligned} \frac{a}{b} &= \frac{c}{d} \\ \therefore a \times d &= b \times c \\ \frac{a \times d}{c \times d} &= \frac{b \times c}{c \times d} \quad \dots(\text{dividing both sides by } c \times d) \\ \frac{a}{c} &= \frac{b}{d} \end{aligned}$$

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$. **This property is known as Alternando.**

(III) Componendo : If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{b} = \frac{c+d}{d}$

$$\frac{a}{b} = \frac{c}{d}$$

$$\frac{a}{b} + 1 = \frac{c}{d} + 1 \quad \dots(\text{adding } 1 \text{ to both sides})$$

$$\frac{a+b}{b} = \frac{c+d}{d}$$

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{b} = \frac{c+d}{d}$. **This property is known as Componendo.**

(IV) Dividendo : If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a-b}{b} = \frac{c-d}{d}$

$$\therefore \frac{a}{b} = \frac{c}{d}$$

$$\therefore \frac{a}{b} - 1 = \frac{c}{d} - 1 \quad \dots(\text{subtracting } 1 \text{ from both sides})$$

$$\therefore \frac{a-b}{b} = \frac{c-d}{d}$$

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a-b}{b} = \frac{c-d}{d}$. **This property is known as Dividendo.**

(V) Componendo-Dividendo : If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$, $a \neq b$, $c \neq d$

If $\frac{a}{b} = \frac{c}{d}$

$$\therefore \frac{a+b}{b} = \frac{c+d}{d} \quad \dots(\text{using componendo}) \quad \dots(1)$$

$$\therefore \frac{a-b}{b} = \frac{c-d}{d} \quad \dots(\text{using dividendo}) \quad \dots(2)$$

$$\therefore \frac{a+b}{a-b} = \frac{c+d}{c-d} \quad \dots\text{from (1) and (2)}$$

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$. **This property is known as Componendo-dividendo.**

General form of Componendo and Dividendo

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{b} = \frac{c+d}{d}$... (performing componendo once)

$$\frac{a+2b}{b} = \frac{c+2d}{d} \quad \dots(\text{performing componendo twice})$$

Generally $\frac{a+mb}{b} = \frac{c+md}{d}$... (performing componendo m times) ... (I)

Similarly if $\frac{a}{b} = \frac{c}{d}$ then $\frac{a-mb}{b} = \frac{c-md}{d}$... (performing dividendo m time) ... (II)

and if $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+mb}{a-mb} = \frac{c+md}{c-md}$... [dividing (I) by (II)]

**Remember this !**

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{b}{a} = \frac{d}{c}$ (Invertendo)

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{b} = \frac{c+d}{d}$ (Componendo)

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{c} = \frac{b}{d}$ (Alternando)

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a-b}{b} = \frac{c-d}{d}$ (Dividendo)

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a+b}{a-b} = \frac{c+d}{c-d}$ (Componendo-Dividendo)

Solved Examples :

Ex (1) If $\frac{a}{b} = \frac{5}{3}$ then find the ratio $\frac{a+7b}{7b} = \dots$

Method I

Solution : If $\frac{a}{b} = \frac{5}{3}$ then $\frac{a}{5} = \frac{b}{3} = k$,
...(using alternando)

$$\therefore a = 5k, b = 3k$$

$$\therefore \frac{a+7b}{7b} = \frac{5k+7 \times 3k}{7 \times 3k}$$

$$= \frac{5k+21k}{21k}$$

$$= \frac{26k}{21k} = \frac{26}{21}$$

Method II

$$\frac{a}{b} = \frac{5}{3}$$

$$\therefore \frac{a}{7b} = \frac{5}{21}$$

$$\therefore \frac{a+7b}{7b} = \frac{5+21}{21} \quad \dots(\text{using componendo})$$

$$\therefore \frac{a+7b}{7b} = \frac{26}{21}$$

Ex. (2) If $\frac{a}{b} = \frac{7}{4}$ then find the ratio $\frac{5a-b}{b} = \dots$

Method I

Solution : $\frac{a}{b} = \frac{7}{4}$

$$\therefore \frac{a}{7} = \frac{b}{4} \quad \dots(\text{using alternando})$$

Let $\frac{a}{7} = \frac{b}{4} = m$

$$\therefore a = 7m, b = 4m$$

$$\therefore \frac{5a-b}{b} = \frac{5(7m) - 4m}{4m}$$

$$= \frac{35m - 4m}{4m}$$

$$= \frac{31}{4}$$

Method II

$$\frac{a}{b} = \frac{7}{4}$$

$$\frac{5a}{b} = \frac{5 \times 7}{4}$$

$$= \frac{35}{4}$$

$$\frac{5a-b}{b} = \frac{35-4}{4} \quad \dots(\text{using dividendo})$$

$$\frac{5a-b}{b} = \frac{31}{4}$$

Ex (5) If $\frac{x}{y} = \frac{4}{5}$ then find the value of the ratio $\frac{4x-y}{4x+y}$.

Solution :

$$\frac{x}{y} = \frac{4}{5}$$

$$\frac{4x}{y} = \frac{16}{5}$$

...(multiplying both sides by 4)

$$\therefore \frac{4x+y}{4x-y} = \frac{16+5}{16-5}$$

...(using componendo-dividendo)

$$\therefore \frac{4x+y}{4x-y} = \frac{21}{11}$$

$$\therefore \frac{4x-y}{4x+y} = \frac{11}{21}$$

∴

Ex (6) If $5x = 4y$ then find the value of the ratio $\frac{3x^2 + y^2}{3x^2 - y^2}$.

Solution :

$$\frac{x}{y} = \frac{4}{5}$$

$$\frac{x^2}{y^2} = \frac{16}{25}$$

$$\therefore \frac{3x^2}{y^2} = \frac{48}{25}$$

...(multiplying both sides by 3)

$$\therefore \frac{3x^2 + y^2}{3x^2 - y^2} = \frac{48+25}{48-25}$$

...(using componendo-dividendo)

$$\therefore \frac{3x^2 + y^2}{3x^2 - y^2} = \frac{73}{23}$$

∴



Let's learn.

Application of properties of equal ratios

To solve some types of equations, it is convenient to use properties of equal ratios rather than using other methods.

Ex (1) Solve the equation. $\frac{3x^2 + 5x + 7}{10x + 14} = \frac{3x^2 + 4x + 3}{8x + 6}$

Solution : $\frac{3x^2 + 5x + 7}{10x + 14} = \frac{3x^2 + 4x + 3}{8x + 6}$

$$\frac{(6x^2 + 10x + 14)}{10x + 14} = \frac{(6x^2 + 8x + 6)}{8x + 6} \quad \dots(\text{multiplying both sides by } 2)$$

$$\frac{(6x^2 + 10x + 14) - (10x + 14)}{10x + 14} = \frac{(6x^2 + 8x + 6) - (8x + 6)}{8x + 6} \quad \dots(\text{using dividendo})$$

$$\therefore \frac{6x^2}{10x + 14} = \frac{6x^2}{8x + 6}$$

This equation is true for $x = 0$ $\therefore x = 0$ is a solution of the given equation.

If $x \neq 0$ then $x^2 \neq 0$, \therefore dividing by $6x^2$, $\frac{1}{10x + 14} = \frac{1}{8x + 6}$

$$\therefore 8x + 6 = 10x + 14$$

$$\therefore 6 - 14 = 10x - 8x$$

$$\therefore -8 = 2x$$

$$\therefore x = -4$$

$\therefore x = -4$ or $x = 0$ are the solutions of the given equation.

Ex (2) Solve. $\frac{\sqrt{x+7} + \sqrt{x-2}}{\sqrt{x+7} - \sqrt{x-2}} = \frac{5}{1}$

Solution : $\frac{(\sqrt{x+7} + \sqrt{x-2}) + (\sqrt{x+7} - \sqrt{x-2})}{(\sqrt{x+7} + \sqrt{x-2}) - (\sqrt{x+7} - \sqrt{x-2})} = \frac{5+1}{5-1} \quad \dots(\text{using componendo-dividendo})$

$$\therefore \frac{2\sqrt{x+7}}{2\sqrt{x-2}} = \frac{6}{4}$$

$$\therefore \frac{\sqrt{x+7}}{\sqrt{x-2}} = \frac{3}{2}$$

$$\therefore \frac{x+7}{x-2} = \frac{9}{4} \quad \dots(\text{squaring both sides of the equation})$$

$$\therefore 4x + 28 = 9x - 18$$

$$\therefore 28 + 18 = 9x - 4x$$

$$\therefore 46 = 5x$$

$$\therefore \frac{46}{5} = x$$

$$\therefore x = \frac{46}{5} \text{ is the solution of the given equation.}$$