

Activity :

Take 5 pieces of card paper. Write the following statements, one on each paper.

(i) $\frac{a+b}{b} = \frac{c+d}{d}$ (ii) $\frac{a}{c} = \frac{b}{d}$ (iii) $\frac{a}{b} = \frac{ac}{bd}$ (iv) $\frac{c}{d} = \frac{c-a}{d-b}$ (v) $\frac{a}{b} = \frac{rc}{rd}$

a, b, c, d are positive numbers and $\frac{a}{b} = \frac{c}{d}$ is given. Which of the above statements are true or false, write at the back of each card, if false explain why.

Practice set 4.3

(1) If $\frac{a}{b} = \frac{7}{3}$ then find the values of the following ratios.

(i) $\frac{5a+3b}{5a-3b}$ (ii) $\frac{2a^2+3b^2}{2a^2-3b^2}$ (iii) $\frac{a^3-b^3}{b^3}$ (iv) $\frac{7a+9b}{7a-9b}$

(2) If $\frac{15a^2+4b^2}{15a^2-4b^2} = \frac{47}{7}$ then find the values of the following ratios.

(i) $\frac{a}{b}$ (ii) $\frac{7a-3b}{7a+3b}$ (iii) $\frac{b^2-2a^2}{b^2+2a^2}$ (iv) $\frac{b^3-2a^3}{b^3+2a^3}$

(3) If $\frac{3a+7b}{3a-7b} = \frac{4}{3}$ then find the value of the ratio $\frac{3a^2-7b^2}{3a^2+7b^2}$.

(4) Solve the following equations.

(i) $\frac{x^2+12x-20}{3x-5} = \frac{x^2+8x+12}{2x+3}$

(ii) $\frac{10x^2+15x+63}{5x^2-25x+12} = \frac{2x+3}{x-5}$

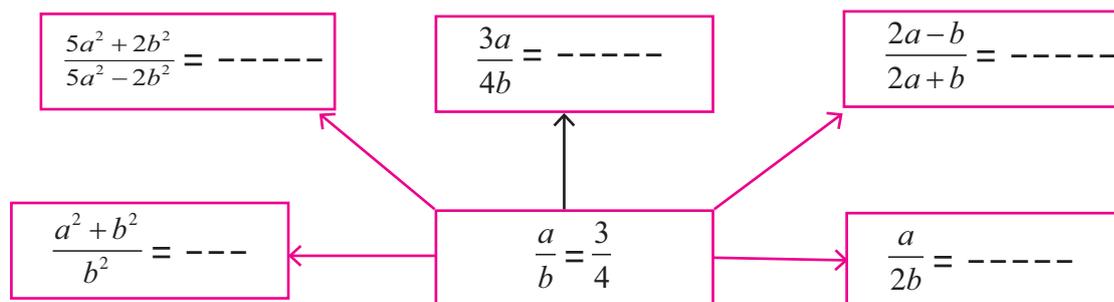
(iii) $\frac{(2x+1)^2+(2x-1)^2}{(2x+1)^2-(2x-1)^2} = \frac{17}{8}$

(iv*) $\frac{\sqrt{4x+1}+\sqrt{x+3}}{\sqrt{4x+1}-\sqrt{x+3}} = \frac{4}{1}$

(v) $\frac{(4x+1)^2+(2x+3)^2}{4x^2+12x+9} = \frac{61}{36}$

(vi) $\frac{(3x-4)^3-(x+1)^3}{(3x-4)^3+(x+1)^3} = \frac{61}{189}$

Activity : In the following activity, the values of a and b can be changed. That is by changing $a : b$ we can create many examples. Teachers should give lot of practice to the students and encourage them to construct their own examples.





Let's learn.

Theorem on equal ratios

If $\frac{a}{b} = \frac{c}{d}$ then $\frac{a}{b} = \frac{a+c}{b+d} = \frac{c}{d}$ This property is called the theorem of equal ratios.

Prrof : Let $\frac{a}{b} = \frac{c}{d} = k$. $\therefore a = bk$ and $c = dk$

$$\therefore \frac{a+c}{b+d} = \frac{bk+dk}{b+d} = \frac{k(b+d)}{b+d} = k$$

$$\therefore \frac{a}{b} = \frac{c}{d} = \frac{a+c}{b+d}$$

We know that, $\frac{a}{b} = \frac{al}{bl}$

$$\therefore \text{If } \frac{a}{b} = \frac{c}{d} = k, \text{ then } \frac{al}{bl} = \frac{cm}{dm} = \frac{al+cm}{bl+dm} = k$$

If $\frac{a}{b} = \frac{c}{d} = \frac{e}{f} = \dots\dots\dots$ (finite terms) and if l, m, n are non-zero numbers

then each ratio = $\frac{al+cm+en+\dots}{bl+dm+fn+\dots}$ (finite terms) is the general form of the above

theorem.



Use your brain power !

In a certain gymnasium, there are 35 girls and 42 boys in the kid's section, 30 girls and 36 boys in the children's section and 20 girls and 24 boys in the teens' section. What is the ratio of the number of boys to the number of girls in every section ?

For physical exercises, all three groups gathered on the ground. Now what is the ratio of number of boys to the number of girls ?

From the answers of the above questions, did you verify the theorem of equal ratios ?

Ex (1) Fill in the blanks in the following statements.

(i) $\frac{a}{3} = \frac{b}{7} = \frac{4a+9b}{\dots\dots\dots}$ (ii) $\frac{x}{3} = \frac{y}{5} = \frac{z}{4} = \frac{5x-3y+4z}{\dots\dots\dots}$

Solution : (i) $\frac{a}{3} = \frac{b}{7} = \frac{4a+9b}{4 \times 3 + 9 \times 7} = \frac{4a+9b}{12+63} = \frac{4a+9b}{75}$

(ii) $\frac{x}{3} = \frac{y}{5} = \frac{z}{4} = \frac{5 \times x}{5 \times 3} = \frac{-3 \times y}{-3 \times 5} = \frac{4 \times z}{4 \times 4}$
 $\therefore = \frac{5x}{15} = \frac{-3y}{-15} = \frac{4z}{16}$
 $= \frac{5x-3y+4z}{15-15+16}$ -----(by the theorem of equal ratio)
 $= \frac{5x-3y+4z}{16}$

If $x = 0$ then $\frac{14x^2 - 6x + 8}{10x^2 + 4x + 7} = \frac{8}{7}$ and $\frac{7x - 3}{5x + 2} = \frac{-3}{2}$

$\therefore \frac{8}{7} = \frac{-3}{2}$ Which is a contradiction.

$\therefore x \neq 0$

\therefore multiplying predecessor and successor of RHS by $2x$.

$$\frac{14x^2 - 6x + 8}{10x^2 + 4x + 7} = \frac{2x(7x - 3)}{2x(5x + 2)} = k$$

$$\therefore \frac{14x^2 - 6x + 8}{10x^2 + 4x + 7} = \frac{14x^2 - 6x}{10x^2 + 4x} = k$$

$$\therefore \frac{14x^2 - 6x + 8 - 14x^2 + 6x}{10x^2 + 4x + 7 - 10x^2 - 4x} = \frac{8}{7} = k$$

$$\therefore k = \frac{8}{7}$$

$$\therefore \frac{7x - 3}{5x + 2} = \frac{8}{7}$$

$$\therefore 49x - 21 = 40x + 16$$

$$\therefore 49x - 40x = 16 + 21$$

$$\therefore 9x = 37 \quad \therefore x = \frac{37}{9}$$

Practice set 4.4

(1) Fill in the blanks of the following

(i) $\frac{x}{7} = \frac{y}{3} = \frac{3x + 5y}{\dots\dots} = \frac{7x - 9y}{\dots\dots}$ (ii) $\frac{a}{3} = \frac{b}{4} = \frac{c}{7} = \frac{a - 2b + 3c}{\dots\dots} = \frac{\dots\dots}{6 - 8 + 14}$

(2) $5m - n = 3m + 4n$ then find the values of the following expressions.

(i) $\frac{m^2 + n^2}{m^2 - n^2}$ (ii) $\frac{3m + 4n}{3m - 4n}$

(3) (i) If $a(y+z) = b(z+x) = c(x+y)$ and out of a, b, c no two of them are equal

then show that, $\frac{y-z}{a(b-c)} = \frac{z-x}{b(c-a)} = \frac{x-y}{c(a-b)}$.

(ii) If $\frac{x}{3x - y - z} = \frac{y}{3y - z - x} = \frac{z}{3z - x - y}$ and $x + y + z \neq 0$ then show that the value of each ratio is equal to 1.

(iii) If $\frac{ax+by}{x+y} = \frac{bx+az}{x+z} = \frac{ay+bz}{y+z}$ and $x+y+z \neq 0$ then show that $\frac{a+b}{2}$.

(iv) If $\frac{y+z}{a} = \frac{z+x}{b} = \frac{x+y}{c}$ then show that $\frac{x}{b+c-a} = \frac{y}{c+a-b} = \frac{z}{a+b-c}$.

(v) If $\frac{3x-5y}{5z+3y} = \frac{x+5z}{y-5x} = \frac{y-z}{x-z}$ then show that every ratio = $\frac{x}{y}$.

(4) Solve. (i) $\frac{16x^2 - 20x + 9}{8x^2 + 12x + 21} = \frac{4x - 5}{2x + 3}$ (ii) $\frac{5y^2 + 40y - 12}{5y + 10y^2 - 4} = \frac{y + 8}{1 + 2y}$



Let's learn.

Continued Proportion

Let us consider the ratios 4 : 12 and 12 : 36. They are equal ratios. In the two ratios, the successor (second term) of the first ratio is equal to the predecessor (first term) of the second ratio. Hence 4, 12, 36 are said to be in continued proportion.

If $\frac{a}{b} = \frac{b}{c}$ then a, b, c are in continued proportion.

If $ac = b^2$, then dividing both sides by bc we get $\frac{a}{b} = \frac{b}{c}$.

\therefore if $ac = b^2$, then a, b, c are in continued proportion.

When a, b, c are in continued proportion then b is known as **Geometric mean** of a and c or **Mean proportional** of a and c .

Hence all the following statements convey the same meaning.

\therefore (1) $\frac{a}{b} = \frac{b}{c}$ (2) $b^2 = ac$ (3) a, b, c are in continued proportion.

(4) b is the geometric mean of a and c .

(5) b is the mean proportional of a and c .

We can generalise the concept of continued proportion.

If $\frac{a}{b} = \frac{b}{c} = \frac{c}{d} = \frac{d}{e} = \frac{e}{f}$ then a, b, c, d, e and f are said to be in continued proportion.

Ex (1) If x is the geometric mean of 25 and 4, then find the value of x .

Solution : x is the geometric mean of 25 and 4.

$$\therefore x^2 = 25 \times 4$$

$$\therefore x^2 = 100$$

$$\therefore x = 10$$

Ex (2) If $4a^2b$, $8ab^2$, p are in continued proportion then find the value of p .

Solution : From given information, $4a^2b$, $8ab^2$, p are in continued proportion.

$$\begin{aligned} \therefore \frac{4a^2b}{8ab^2} &= \frac{8ab^2}{p} \\ p &= \frac{8ab^2 \times 8ab^2}{4a^2b} = 16b^3 \end{aligned}$$

Ex (3) Which number should be subtracted from 7, 12 and 18 such that the resultant numbers are in continued proportion?

Solution : Let x be subtracted from 7, 12 and 18 such that resultant numbers are in continued proportion.

| | | |
|--|--|---|
| $(7-x), (12-x), (18-x)$ are in continued proportion. $\therefore (12-x)^2 = (7-x)(18-x)$ $\therefore 144 - 24x + x^2 = 126 - 25x + x^2$ $\therefore -24x + 25x = 126 - 144$ $\therefore x = -18$ | | <p style="text-align: right;">Tally</p> $(7-x) = 7 - (-18) = 25$ $(12-x) = 12 - (-18) = 30$ $(18-x) = 18 - (-18) = 36$ $30^2 = 900$ and $25 \times 36 = 900$ 25, 30, 36 are in continued proportion |
|--|--|---|

\therefore If -18 is subtracted from 7, 12, 18 the resultant numbers are in continued proportion.

k - method

The k -method is used to solve examples based on equal ratios, i.e. equal proportions.

In this simple method every equal ratio is assumed to be equal to k .

Ex (1) If $\frac{a}{b} = \frac{c}{d}$ then show that $\frac{5a-3c}{5b-3d} = \frac{7a-2c}{7b-2d}$

Solution : Let $\frac{a}{b} = \frac{c}{d} = k \quad \therefore a = bk, c = dk$

Substituting values of a and c in both sides,

$$\begin{aligned} \text{LHS} &= \frac{5a-3c}{5b-3d} = \frac{5(bk)-3(dk)}{5b-3d} = \frac{k(5b-3d)}{(5b-3d)} = k \\ \text{RHS} &= \frac{7a-2c}{7b-2d} = \frac{7(bk)-2(dk)}{7b-2d} = \frac{k(7b-2d)}{7b-2d} = k \\ \therefore \text{LHS} &= \text{RHS.} \end{aligned}$$

$$\therefore \frac{5a-3c}{5b-3d} = \frac{7a-2c}{7b-2d}$$

Ex (2) If a, b, c are in continued proportion then show that, $\frac{(a+b)^2}{ab} = \frac{(b+c)^2}{bc}$.

Solution : a, b, c are in continued proportion. Let $\frac{a}{b} = \frac{b}{c} = k$.

$$\therefore b = ck, \quad a = bk = ck \times k = ck^2$$

Substituting values of a and b .

$$\text{LHS} = \frac{(a+b)^2}{ab} = \frac{(ck^2 + ck)^2}{(ck^2)(ck)} = \frac{c^2k^2(k+1)^2}{c^2k^3} = \frac{(k+1)^2}{k}$$

$$\text{RHS} = \frac{(b+c)^2}{bc} = \frac{(ck + c)^2}{(ck)c} = \frac{c^2(k+1)^2}{c^2k} = \frac{(k+1)^2}{k}$$

$$\therefore \text{LHS} = \text{RHS}. \quad \therefore \frac{(a+b)^2}{ab} = \frac{(b+c)^2}{bc}$$

Ex (3) If a, b, c are in continued proportion

then show that $\frac{a}{c} = \frac{a^2 + ab + b^2}{b^2 + bc + c^2}$

Solution : a, b, c are in continued proportion.

$$\therefore \frac{a}{b} = \frac{b}{c}$$

$$\text{Let, } \frac{a}{b} = \frac{b}{c} = k \quad \therefore b = ck \quad \text{and } a = ck^2$$

$$\text{LHS} = \frac{a}{c} = \frac{ck^2}{c} = k^2$$

$$\begin{aligned} \text{RHS} &= \frac{a^2 + ab + b^2}{b^2 + bc + c^2} \\ &= \frac{(k^2c)^2 + k^2c(ck) + (ck)^2}{(ck)^2 + (ck)(c) + c^2} \\ &= \frac{k^4c^2 + k^3c^2 + c^2k^2}{c^2k^2 + c^2k + c^2} \\ &= \frac{c^2k^2(k^2 + k + 1)}{c^2(k^2 + k + 1)} \\ &= k^2 \end{aligned}$$

$$\therefore \text{LHS} = \text{RHS}$$

$$\therefore \frac{a}{c} = \frac{a^2 + ab + b^2}{b^2 + bc + c^2}$$

Ex (4) Five numbers are in continued proportion. The first term is 5 and the last term is 80. Find these numbers.

Solution : Let the numbers in continued proportion be a, ak, ak^2, ak^3, ak^4 .

$$\text{Here } a = 5 \quad \text{and } ak^4 = 80$$

$$\therefore 5 \times k^4 = 80$$

$$\therefore k^4 = 16$$

$$\therefore k = 2 \quad \because 2^4 = 16$$

$$ak = 5 \times 2 = 10 \quad ak^2 = 5 \times 4 = 20$$

$$ak^3 = 5 \times 8 = 40 \quad ak^4 = 5 \times 16 = 80$$

\therefore the numbers are 5, 10, 20, 40, 80.

Practice set 4.5

- (1) Which number should be subtracted from 12, 16 and 21 so that resultant numbers are in continued proportion?
- (2) If $(28-x)$ is the mean proportional of $(23-x)$ and $(19-x)$ then find the value of x .
- (3) Three numbers are in continued proportion, whose mean proportional is 12 and the sum of the remaining two numbers is 26, then find these numbers.
- (4) If $(a + b + c)(a - b + c) = a^2 + b^2 + c^2$ show that a, b, c are in continued proportion.
- (5) If $\frac{a}{b} = \frac{b}{c}$ and $a, b, c > 0$ then show that,
 - (i) $(a + b + c)(b - c) = ab - c^2$
 - (ii) $(a^2 + b^2)(b^2 + c^2) = (ab + bc)^2$
 - (iii) $\frac{a^2 + b^2}{ab} = \frac{a + c}{b}$
- (6) Find mean proportional of $\frac{x + y}{x - y}, \frac{x^2 - y^2}{x^2 y^2}$

Activity : Observe the political map of India from a Geography textbook. Study the scale of this map.

From the given scale find the straight line distances between various cities like

- (i) New Delhi to Bengaluru (ii) Mumbai to Kolkata, (iii) Jaipur to Bhubaneshwar.

Problem set 4

- (1) Select the appropriate alternative answer for the following questions.
 - (i) If $6 : 5 = y : 20$ then what will be the value of y ?
(A) 15 (B) 24 (C) 18 (D) 22.5
 - (ii) What is the ratio of 1 mm to 1 cm ?
(A) 1 : 100 (B) 10 : 1 (C) 1 : 10 (D) 100 : 1
 - (iii*) The ages of Jatin, Nitin and Mohasin are 16, 24 and 36 years respectively. What is the ratio of Nitin's age to Mohasin's age ?
(A) 3 : 2 (B) 2 : 3 (C) 4 : 3 (D) 3 : 4

- (iv) 24 Bananas were distributed between Shubham and Anil in the ratio 3 : 5, then how many bananas did Shubham get ?
 (A) 8 (B) 15 (C) 12 (D) 9
- (v) What is the mean proportional of 4 and 25 ?
 (A) 6 (B) 8 (C) 10 (D) 12
- (2) For the following numbers write the ratio of first number to second number in the reduced form.
- (i) 21, 48 (ii) 36, 90 (iii) 65, 117 (iv) 138, 161 (v) 114, 133
- (3) Write the following ratios in the reduced form.
- (i) Radius to the diameter of a circle.
 (ii) The ratio of diagonal to the length of a rectangle, having length 4 cm and breadth 3 cm.
 (iii) The ratio of perimeter to area of a square, having side 4 cm.
- (4) Check whether the following numbers are in continued proportion.
- (i) 2, 4, 8 (ii) 1, 2, 3 (iii) 9, 12, 16 (iv) 3, 5, 8
- (5) a, b, c are in continued proportion. If $a = 3$ and $c = 27$ then find b .
- (6) Convert the following ratios into percentages..
- (i) $37 : 500$ (ii) $\frac{5}{8}$ (iii) $\frac{22}{30}$ (iv) $\frac{5}{16}$ (v) $\frac{144}{1200}$
- (7) Write the ratio of first quantity to second quantity in the reduced form.
- (i) 1024 MB, 1.2 GB [(1024 MB = 1 GB)]
 (ii) 17 Rupees, 25 Rupees 60 paise (iii) 5 dozen, 120 units
 (iv) 4 sq.m, 800 sq.cm (v) 1.5 kg, 2500 gm
- (8) If $\frac{a}{b} = \frac{2}{3}$ then find the values of the following expressions.
- (i) $\frac{4a+3b}{3b}$ (ii) $\frac{5a^2+2b^2}{5a^2-2b^2}$
 (iii) $\frac{a^3+b^3}{b^3}$ (iv) $\frac{7b-4a}{7b+4a}$
- (9) If a, b, c, d are in proportion, then prove that
- (i) $\frac{11a^2+9ac}{11b^2+9bd} = \frac{a^2+3ac}{b^2+3bd}$
 (ii*) $\sqrt{\frac{a^2+5c^2}{b^2+5d^2}} = \frac{a}{b}$
 (iii) $\frac{a^2+ab+b^2}{a^2-ab+b^2} = \frac{c^2+cd+d^2}{c^2-cd+d^2}$

(10) If a, b, c are in continued proportion, then prove that

$$(i) \quad \frac{a}{a+2b} = \frac{a-2b}{a-4c} \quad (ii) \quad \frac{b}{b+c} = \frac{a-b}{a-c}$$

(11) Solve : $\frac{12x^2 + 18x + 42}{18x^2 + 12x + 58} = \frac{2x + 3}{3x + 2}$

(12) If $\frac{2x-3y}{3z+y} = \frac{z-y}{z-x} = \frac{x+3z}{2y-3x}$ then prove that every ratio = $\frac{x}{y}$.

(13*) If $\frac{by+cz}{b^2+c^2} = \frac{cz+ax}{c^2+a^2} = \frac{ax+by}{a^2+b^2}$ then prove that $\frac{x}{a} = \frac{y}{b} = \frac{z}{c}$.



5

Linear Equations in Two Variables



Let's study.

- Introduction
- Linear equation in two variables
- Solving simultaneous equations
- Simultaneous equations
- Word problems based on simultaneous equations



Let's recall.

Ex. Solve the following equations.

(1) $m + 3 = 5$

$m = \square$

(2) $3y + 8 = 22$

$y = \square$

(3) $\frac{x}{3} = 2$

$x = \square$

(4) $2p = p + \frac{4}{9}$

$p = \square$

(5) Which number should be added to 5 to obtain 14 ?

$\square + 5 = 14$

$x + 5 = 14$

$x = \square$

(6) Which number should be subtracted from 8 to obtain 2 ?

$8 - \square = 2$

$8 - y = 2$

$y = \square$

In all above equations, degree of the variable is 1. These are called as Linear equations.



Let's learn.

Linear equations in two variables

Find two numbers whose sum is 14.

Using variables x and y for the two numbers, we can form the equation $x + y = 14$.

This is an equation in two variables.

We can find many values of x and y satisfying the condition.

e.g. $9 + 5 = 14$

$7 + 7 = 14$

$8 + 6 = 14$

$4 + 10 = 14$

$(-1) + 15 = 14$

$15 + (-1) = 14$

$2.6 + 11.4 = 14$

$0 + 14 = 14$

$100 + (-86) = 14$

$(-100) + (114) = 14$

$\square + \square = 14$

$\square + \square = 14$

Hence, above equation has many solutions like $(x = 9, y = 5)$; $(x = 7, y = 7)$; $(x = 8, y = 6)$ etc.

Conventionally, the solution $x = 9, y = 5$ is written as an ordered pair $(9, 5)$ where 9 is the value of x and 5 is the value of y . To satisfy the equation $x + y = 14$, we can get infinite ordered pairs like $(9,5), (7,7), (8,6), (4,10), (10,4), (-1,15), (2.6, 11.4), \dots$ etc. All of these are the solutions of $x + y = 14$.

Consider second example.

Find two numbers such that their difference is 2.

Let the greater number be x and the smaller number be y .

Then we get the equation $x - y = 2$

For the values of x and y , we can get following equations.

$$10 - 8 = 2 \quad 9 - 7 = 2 \quad 8 - 6 = 2 \quad (-3) - (-5) = 2 \quad 5.3 - 3.3 = 2$$

$$15 - 13 = 2 \quad 100 - 98 = 2 \quad \square - \square = 2 \quad \square - \square = 2$$

Here if we take values $x = 10$ and $y = 8$, then the ordered pair $(10, 8)$ satisfies the above equation. Here we cannot write as $(8, 10)$ because $(8, 10)$ will imply $x = 8$ and $y = 10$ and it does not satisfy the equation $x - y = 2$. Therefore, note that, the order of numbers in the pair indicating solution is very important.

Now let us write the solutions of $x - y = 2$ in the form of ordered pairs.

$(7, 5), (-2, -4), (0, -2), (5.2, 3.2), (8, 6)$ etc. There are infinite solutions.

Find the solution of $4m - 3n = 2$.

Construct 3 different equations and find their solutions.

Now, observe the first two equations.

$$x + y = 14 \quad \dots\dots\dots \text{I}$$

$$x - y = 2 \quad \dots\dots\dots \text{II}$$

Solution of equation I : $(9, 5), (7, 7), (8, 6)\dots$

Solutions of Equation II : $(7, 5), (-2, -4), (0, -2), (5.2, 3.2), (8, 6)\dots$

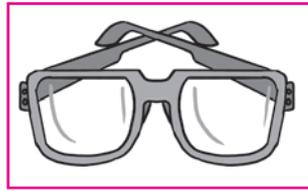
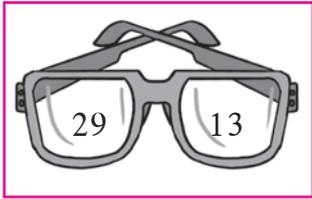
$(8, 6)$ is the only common solution of both the equations. This solution satisfies both the equations. Hence it is the unique common solution of both the equations.



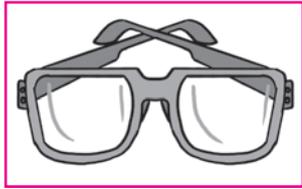
Remember this !

When we consider at the same time two linear equations in two variables those equations are called **Simultaneous equations**.

Activity : On the glasses of following spectacles, write numbers such that



(i) Their sum is 42 and difference is 16 (ii) Their sum is 37 and difference is 11



(iii) Their sum is 54 and difference is 20 (iv) Their sum is ... and difference is ...



Let's recall.

$x+y = 5$ and $2x + 2y = 10$ are two equations in two variables. Find five different solutions of $x+y = 5$, verify whether same solutions satisfy the equation $2x + 2y = 10$ also.

Observe both equations.

Find the condition where two equations in two variables have all solutions in common.



Let's learn.

Elimination method of solving simultaneous equations

By taking different values of variables we have solved the equations $x + y = 14$ and $x - y = 2$. But every time, it is not easy to solve by this method, e.g. , $2x + 3y = -4$ and $x - 5y = 11$. Try to solve these equations by taking different values of x and y . By this method observe that it is not easy to obtain the solution.

Therefore to solve simultaneous equations we use different method. In this method, we eliminate one of the variables to obtain equations in one variable. We can solve and find the value of one of the two variables and then substituting this value in one of the given equations we can find the value of the other variable.

Study the following example to understand this method.

Ex (1) Solve $x + y = 14$ and $x - y = 2$.

Solution : By adding both the equations we get an equation in one variable

$$\begin{array}{rcl} x + y & = & 14 \quad \text{.....I} \\ + \quad x - y & = & 2 \quad \text{.....II} \\ \hline 2x + 0 & = & 16 \\ 2x & = & 16 \\ x & = & 8 \end{array} \quad \left| \begin{array}{l} \text{Substituting } x=8 \text{ in the equation (I)} \\ x + y = 14 \\ \therefore 8 + y = 14 \\ \therefore y = 6 \end{array} \right.$$

Here (8, 6) is the solution of first equation. Let us check, whether it satisfies the second equation also.

$$x - y = 8 - 6 = 2 \text{ is true.}$$

\therefore (8,6) is the solution for both the equations.

Hence (8, 6) is the solution of simultaneous equations $x + y = 14$ and $x - y = 2$.

Ex (2) Sum of the ages of mother and son is 45 years. If son's age is subtracted from twice of mother's age then we get answer 54. Find the ages of mother and son.

It becomes easy to solve a problem if we make use of variables.

Solution : Let the mother's today's age be x years and son's today's age be y years.

$$\text{From the first condition } x+y=45 \quad \text{.....I}$$

$$\text{From the second condition } 2x-y = 54 \quad \text{.....II}$$

$$\text{Adding equations (I) and (II)} \quad 3x+0 = 99$$

$$3x = 99$$

$$x = 33$$

$$\text{Substituting } x = 33 \text{ in equation (I), } 33+y = 45$$

$$y = 45-33$$

$$y = 12$$

Verify that $x=33$ and $y = 12$ is the solution of second equation.

Today's age of mother = 33 and today's age of son = 12.

General form of linear equation in two variables

The general form of a linear equation in two variables is $ax + by + c = 0$ where a, b, c are real numbers and a and b are non-zero at the same time.

In this equation the index of both the variables is 1. Hence it is a linear equation.

Ex (1) Solve the following equations

$$3x + y = 5 \dots\dots\dots (I)$$

$$2x + 3y = 1 \dots\dots\dots (II)$$

Solution : To eliminate one of the variables, we observe that in both equations, not a single coefficient is equal or opposite number. Hence we will make one of them equal.

Multiply both sides of the equation (I) by 3.

$$\therefore 3x \times 3 + 3 \times y = 5 \times 3$$

$$\therefore 9x + 3y = 15 \dots\dots\dots (III)$$

$$2x + 3y = 1 \dots\dots\dots (II)$$

Now subtracting eqn (II) from eqn (III)

$$\begin{array}{r} 9x + 3y = 15 \\ + 2x + 3y = 1 \\ \hline 7x = 14 \\ x = 2 \end{array}$$

Substituting $x = 2$ in one of the equations.

$$\begin{aligned} 2x + 3y &= 1 \\ \therefore 2 \times 2 + 3y &= 1 \\ \therefore 4 + 3y &= 1 \\ \therefore 3y &= -3 \\ \therefore y &= -1 \end{aligned}$$

Verify that $(2, -1)$ satisfies the second equation.

Ex (2) Solve the following simultaneous equations.

$$3x - 4y - 15 = 0 \dots\dots\dots (I)$$

$$y + x + 2 = 0 \dots\dots\dots (II)$$

Solution : Let us write the equations by shifting constant terms to RHS

$$3x - 4y = 15 \dots\dots\dots (I)$$

$$x + y = -2 \dots\dots\dots (II)$$

To eliminate y , multiply second equation by 4 and add to equation (I).

$$\begin{array}{r} 3x - 4y = 15 \\ + 4x + 4y = -8 \\ \hline 7x = 7 \\ x = 1 \end{array}$$

Substituting $x = 1$ in the equation (II).

$$\begin{aligned} x + y &= -2 \\ \therefore 1 + y &= -2 \\ \therefore y &= -2 - 1 \\ \therefore y &= -3 \end{aligned}$$

$(1, -3)$ is the solution of the above equations.

Verify that it satisfies equation (I) also.



Use your brain power!

$3x - 4y - 15 = 0$ and $y + x + 2 = 0$. Can these equations be solved by eliminating x ? Is the solution same?