



Let's study.

- Theorem of remote interior angles of a triangle
- Congruence of triangles
- Theorem of an isosceles triangle
- Property of 30° - 60° - 90° angled triangle
- Median of a triangle
- Property of median on hypotenuse of a right angled triangle
- Perpendicular bisector theorem
- Angle bisector theorem
- Similar triangles

Activity :

Draw a triangle of any measure on a thick paper. Take a point T on ray QR as shown in fig. 3.1. Cut two pieces of thick paper which will exactly fit the corners of $\angle P$ and $\angle Q$. See that the same two pieces fit exactly at the corner of $\angle PRT$ as shown in the figure.

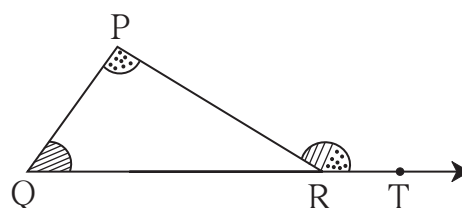


Fig. 3.1



Let's learn.

Theorem of remote interior angles of a triangle

Theorem : The measure of an exterior angle of a triangle is equal to the sum of its remote interior angles.

Given : $\angle PRS$ is an exterior angle of ΔPQR .

To prove : $\angle PRS = \angle PQR + \angle QPR$

Proof : The sum of all angles of a triangle is 180° .

$$\therefore \angle PQR + \angle QPR + \angle PRQ = 180^\circ \dots\dots(I)$$

$$\angle PRQ + \angle PRS = 180^\circ \dots\dots\text{angles in linear pair} \dots\dots(II)$$

$\therefore$ from (I) and (II)

$$\angle PQR + \angle QPR + \angle PRQ = \angle PRQ + \angle PRS$$

$$\therefore \angle PQR + \angle QPR = \angle PRS \dots\dots\text{eliminating } \angle PRQ \text{ from both sides}$$

$\therefore$ the measure of an exterior angle of a triangle is equal to the sum of its remote interior angles.

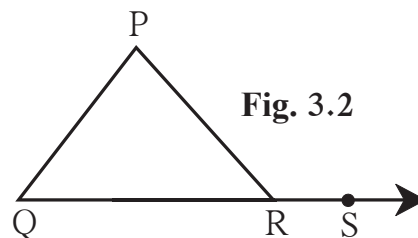
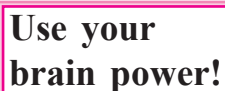


Fig. 3.2





Can we give an alternative proof of the theorem drawing a line through point R and parallel to seg PQ in figure 3.2 ?



Property of an exterior angle of triangle

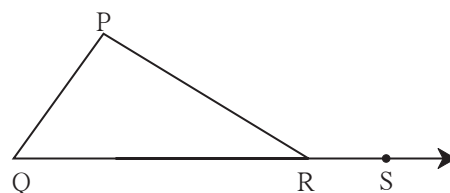
The sum of two positive numbers a and b , that is $(a + b)$ is greater than a and greater than b also. That is, $a + b > a$, $a + b > b$

Using this inequality we get one property related to exterior angle of a triangle.

If $\angle PRS$ is an exterior angle of ΔPQR then

$$\angle PRS > \angle P, \quad \angle PRS > \angle Q$$

$\therefore$ an exterior angle of a triangle is greater than its remote interior angle.



Solved examples

Ex (1) The measures of angles of a triangle are in the ratio 5 : 6 : 7. Find the measures.

Solution : Let the measures of the angles of a triangle be $5x$, $6x$, $7x$.

$$\therefore 5x + 6x + 7x = 180^\circ$$

$$18x = 180^\circ$$

$x = 10^\circ$

$$5x = 5 \times 10 = 50^\circ, \quad 6x = 6 \times 10 = 60^\circ, \quad 7x = 7 \times 10 = 70^\circ$$

$\therefore$ the measures of angles of the triangle are 50° , 60° and 70° .

Ex (2) Observe figure 3.4 and find the measures of $\angle PRS$ and $\angle RTS$.

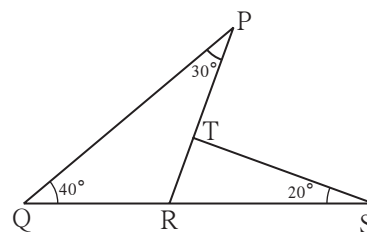
Solution : $\angle$ PRS is an exterior angle of Δ PQR.

So from the theorem of remote interior angles,

$$\angle PRS = \angle PQR + \angle QPR$$

$$= 40^\circ + 30^\circ$$

$$= 70^\circ$$



In Δ RTS

$$\angle TRS + \angle RTS + \angle TSR = \boxed{} \text{ sum of all angles of a triangle}$$

$$\therefore \boxed{} + \angle \text{RTS} + \boxed{} = 180^\circ$$

$$\therefore \angle \text{RTS} + 90^\circ = 180^\circ$$

$$\therefore \angle \text{RTS} = \boxed{}$$

Ex (3) Prove that the sum of exterior angles of a triangle, obtained by extending its sides in the same direction is 360° .

Given : $\angle PAB, \angle QBC$ and $\angle ACR$
are exterior angles of $\triangle ABC$

To prove : $\angle PAB + \angle QBC + \angle ACR = 360^\circ$

Proof : **Method I**

Considering exterior $\angle PAB$ of $\triangle ABC$,
 $\angle ABC$ and $\angle ACB$ are its remote interior angles.

$$\angle PAB = \angle ABC + \angle ACB \text{ ----(I)}$$

Similarly, $\angle ACR = \angle ABC + \angle BAC$ ----(II)..theorem of remote interior angles
and $\angle CBQ = \angle BAC + \angle ACB$ ---- (III)

Adding (I), (II) and (III),

$$\begin{aligned} \angle PAB + \angle ACR + \angle CBQ &= \angle ABC + \angle ACB + \angle ABC + \angle BAC + \angle BAC + \angle ACB \\ &= 2\angle ABC + 2\angle ACB + 2\angle BAC \\ &= 2(\angle ABC + \angle ACB + \angle BAC) \\ &= 2 \times 180^\circ \dots\dots \text{sum of interior angles of a triangle} \\ &= 360^\circ \end{aligned}$$

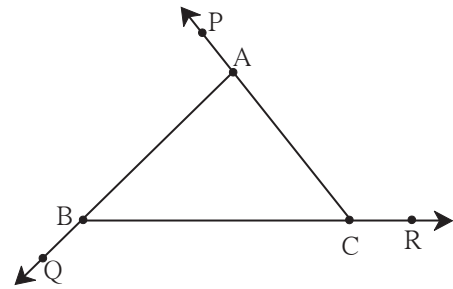


Fig. 3.5

Method II

$$\angle c + \angle f = 180^\circ \dots\dots (\text{angles in linear pair})$$

$$\text{Also, } \angle a + \angle d = 180^\circ$$

$$\text{and } \angle b + \angle e = 180^\circ$$

$$\therefore \angle c + \angle f + \angle a + \angle d + \angle b + \angle e = 180^\circ \times 3 = 540^\circ$$

$$\angle f + \angle d + \angle e + (\angle a + \angle b + \angle c) = 540^\circ$$

$$\therefore \angle f + \angle d + \angle e + 180^\circ = 540^\circ$$

$$\therefore f + d + e = 540^\circ - 180^\circ$$

$$= 360^\circ$$

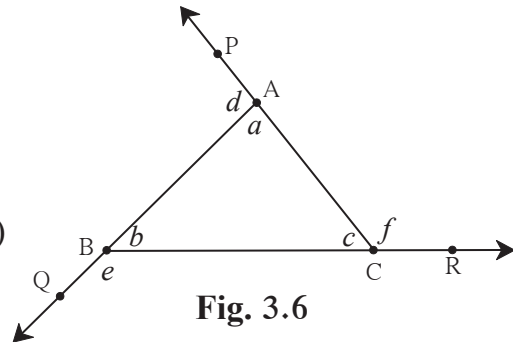


Fig. 3.6

Ex (4) In figure 3.7, bisectors of $\angle B$ and $\angle C$ of $\triangle ABC$ intersect at point P.

Prove that $\angle BPC = 90^\circ + \frac{1}{2} \angle BAC$.

Complete the proof filling in the blanks.

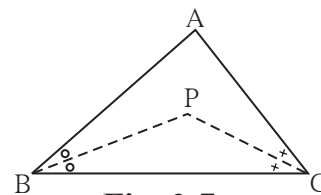


Fig. 3.7

Proof : In $\triangle ABC$,

$\angle BAC + \angle ABC + \angle ACB = \square$ sum of measures of angles of a triangle

$$\therefore \frac{1}{2} \angle BAC + \frac{1}{2} \angle ABC + \frac{1}{2} \angle ACB = \frac{1}{2} \times \square$$

....multiplying each term by $\frac{1}{2}$

$$\therefore \frac{1}{2} \angle BAC + \angle PBC + \angle PCB = 90^\circ$$

$$\therefore \angle PBC + \angle PCB = 90^\circ - \frac{1}{2} \angle BAC \text{(I)}$$

In $\triangle BPC$

$\angle BPC + \angle PBC + \angle PCB = 180^\circ$ sum of measures of angles of a triangle

$$\therefore \angle BPC + \square = 180^\circ \text{from (I)}$$

$$\therefore \angle BPC = 180^\circ - (90^\circ - \frac{1}{2} \angle BAC)$$

$$= 180^\circ - 90^\circ + \frac{1}{2} \angle BAC$$

$$= 90^\circ + \frac{1}{2} \angle BAC$$

Practice set 3.1

1. In figure 3.8, $\angle ACD$ is an exterior angle of $\triangle ABC$.

$\angle B = 40^\circ$, $\angle A = 70^\circ$. Find the measure of $\angle ACD$.

2. In $\triangle PQR$, $\angle P = 70^\circ$, $\angle Q = 65^\circ$ then find $\angle R$.

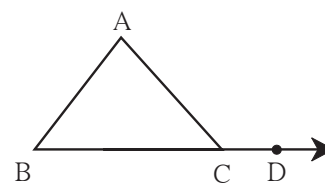


Fig. 3.8

3. The measures of angles of a triangle are x° , $(x-20)^\circ$, $(x-40)^\circ$.

Find the measure of each angle.

4. The measure of one of the angles of a triangle is twice the measure of its smallest angle and the measure of the other is thrice the measure of the smallest angle. Find the measures of the three angles.

5. In figure 3.9, measures of some angles are given. Using the measures find the values of x, y, z .

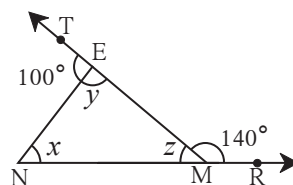


Fig. 3.9

- 6.** In figure 3.10, line $AB \parallel$ line DE . Find the measures of $\angle DRE$ and $\angle ARE$ using given measures of some angles.

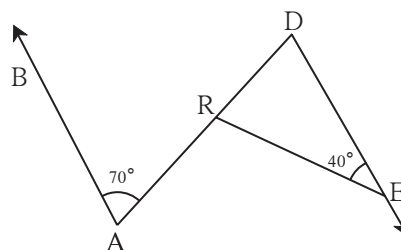


Fig. 3.10

7. In $\triangle ABC$, bisectors of $\angle A$ and $\angle B$ intersect at point O. If $\angle C = 70^\circ$. Find measure of $\angle AOB$.

- 8.** In Figure 3.11, line $AB \parallel$ line CD and line PQ is the transversal. Ray PT and ray QT are bisectors of $\angle BPQ$ and $\angle PQD$ respectively.
Prove that $m\angle PTQ = 90^\circ$.

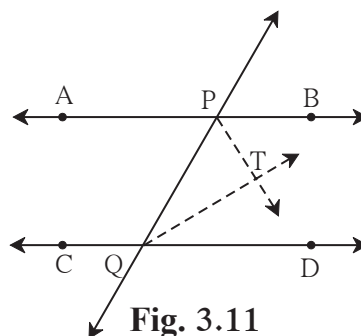


Fig. 3.11

9. Using the information in figure 3.12, find the measures of $\angle a$, $\angle b$ and $\angle c$.

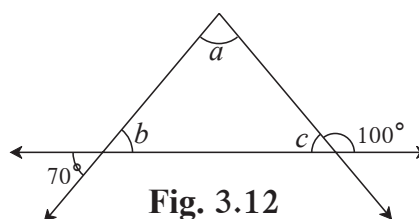


Fig. 3.12

- 10.** In figure 3.13, line $DE \parallel$ line GF
ray EG and ray FG are bisectors of
 $\angle DEF$ and $\angle DFM$ respectively.
Prove that,
(i) $\angle DEG = \frac{1}{2} \angle EDF$ (ii) $EF = FG$.

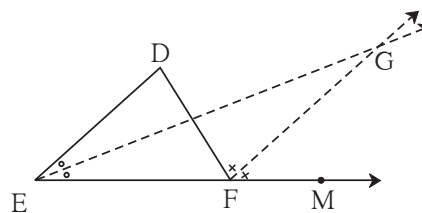


Fig. 3.13



Let's learn.

Congruence of triangles

We know that, if a segment placed upon another fits with it exactly then the two segments are congruent. When an angle placed upon another fits with it exactly then the two angles are congruent. Similarly, if a triangle placed upon another triangle fits exactly with it then the two triangles are said to be congruent. If $\triangle ABC$ and $\triangle PQR$ are congruent is written as $\triangle ABC \cong \triangle PQR$.

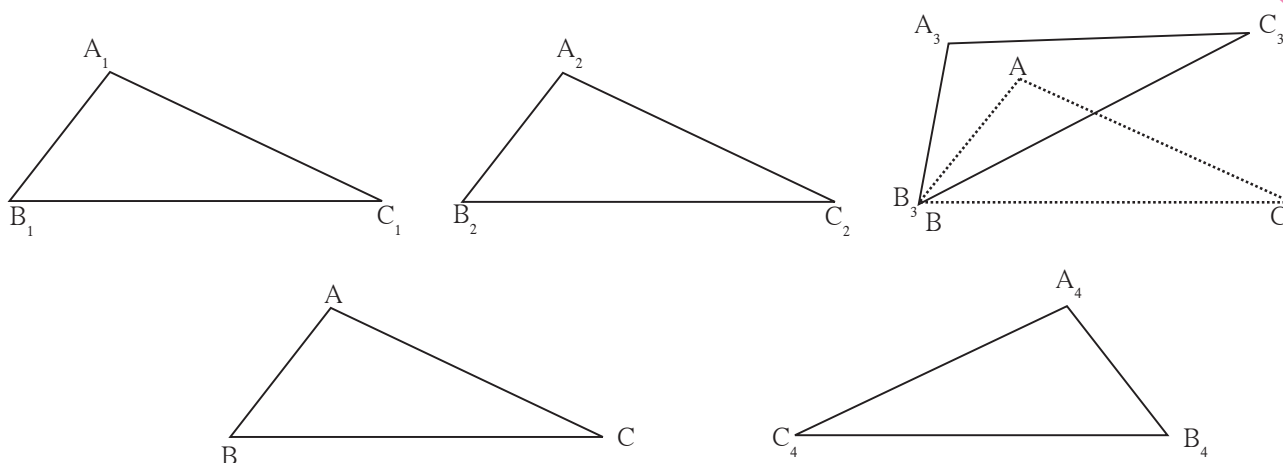


Fig. 3.14

Activity : Draw $\triangle ABC$ of any measure on a card-sheet and cut it out.

Place it on a card-sheet. Make a copy of it by drawing its border. Name it as $\triangle A_1B_1C_1$.

Now slide the $\triangle ABC$ which is the cut out of a triangle to some distance and make one more copy of it. Name it $\triangle A_2B_2C_2$.

Then rotate the cut out of triangle ABC a little, as shown in the figure, and make another copy of it. Name the copy as $\triangle A_3B_3C_3$. Then flip the triangle ABC , place it on another card-sheet and make a new copy of it. Name this copy as $\triangle A_4B_4C_4$.

Have you noticed that each of $\triangle A_1B_1C_1$, $\triangle A_2B_2C_2$, $\triangle A_3B_3C_3$ and $\triangle A_4B_4C_4$ is congruent with $\triangle ABC$? Because each of them fits exactly with $\triangle ABC$.

Let us verify for $\triangle A_3B_3C_3$. If we place $\angle A$ upon $\angle A_3$, $\angle B$ upon $\angle B_3$ and $\angle C$ upon $\angle C_3$, then only they will fit each other and we can say that $\triangle ABC \cong \triangle A_3B_3C_3$.

We also have $AB = A_3B_3$, $BC = B_3C_3$, $CA = C_3A_3$.

Note that, while examining the congruence of two triangles, we have to write their angles and sides in a specific order, that is with a specific one-to-one correspondence.

If $\triangle ABC \cong \triangle PQR$, then we get the following six equations :

$\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$. . . (I) and $AB = PQ$, $BC = QR$, $CA = RP$. . . (II)

This means, with a one-to-one correspondence between the angles and the sides of two triangles, we get three pairs of congruent angles and three pairs of congruent sides.

Given six equations above are true for congruent triangles. For this let us see three specific equations are true then all six equations become true and hence two triangles congruent.

- (1) In a correspondence, if two angles of $\triangle ABC$ are equal to two angles of $\triangle PQR$ and the sides included by the respective pairs of angles are also equal, then the two triangles are congruent.

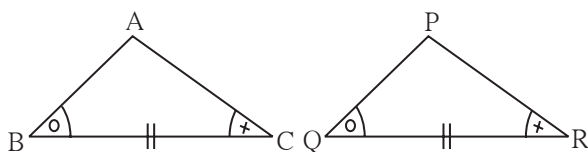


Fig. 3.15

This property is called as angle-side-angle test, which in short we write A-S-A test.

- (2) In a correspondence, if two sides of $\triangle ABC$ are equal to two sides of $\triangle PQR$ and the angles included by the respective pairs of sides are also equal, then the two triangles are congruent.

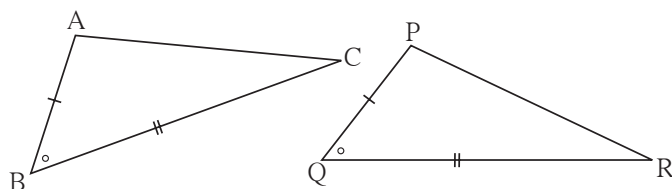


Fig. 3.16

This property is called as side-angle-side test, which in short we write S-A-S test.

- (3) In a correspondence, if three sides of $\triangle ABC$ are equal to three sides of $\triangle PQR$, then the two triangles are congruent.

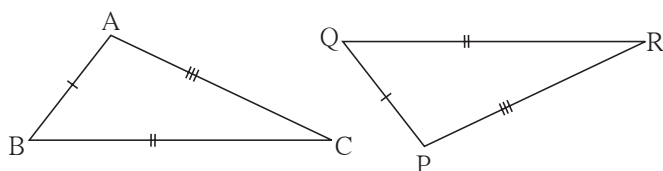


Fig. 3.17

This property is called as side-side-side test, which in short we write S-S-S test.

- (4) If in $\triangle ABC$ and $\triangle PQR$, $\angle B$ and $\angle Q$ are right angles, hypotenuses are equal and $AB = PQ$, then the two triangles are congruent.

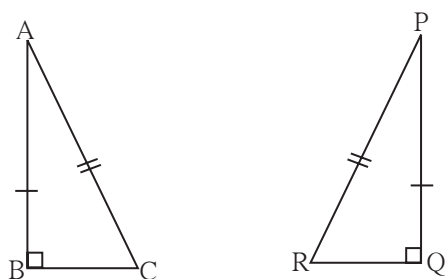


Fig. 3.18

This property is called the hypotenuse side test.



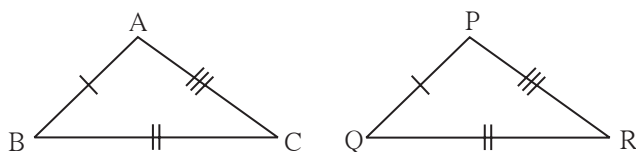
Remember this !

We have constructed triangles using the given information about parts of triangles. (For example, two angles and the included side, three sides, two sides and an included angle). We have experienced that the triangle constructed with any of these information is unique. So if by some one-to-one correspondence between two triangles, these three parts of one triangle are congruent with corresponding three parts of the other triangle then the two triangles are congruent. Then we come to know that in that correspondence their three angles and three sides are congruent. If two triangles are congruent then their respective angles and respective sides are congruent. This property is useful to solve many problems in Geometry.

Practice set 3.2

1. In each of the examples given below, a pair of triangles is shown. Equal parts of triangles in each pair are marked with the same signs. Observe the figures and state the test by which the triangles in each pair are congruent.

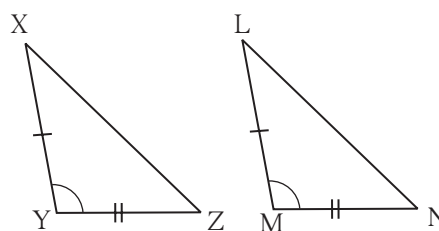
(i)



By test

$$\triangle ABC \cong \triangle PQR$$

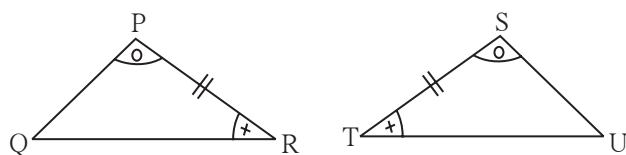
(ii)



By test

$$\triangle XYZ \cong \triangle LMN$$

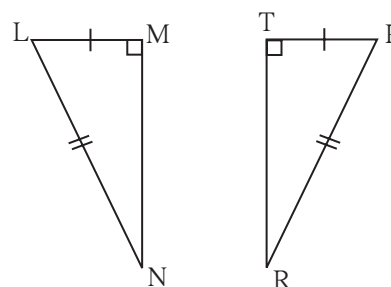
(iii)



By test

$$\triangle PRQ \cong \triangle STU$$

(iv)



By test

$$\triangle LMN \cong \triangle PTR$$

Fig. 3.19

2. Observe the information shown in pairs of triangles given below. State the test by which the two triangles are congruent. Write the remaining congruent parts of the triangles.

(i)

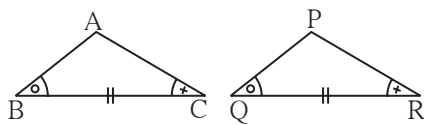


Fig. 3.20

From the information shown in the figure,
in $\triangle ABC$ and $\triangle PQR$

$$\angle ABC \cong \angle PQR$$

$$\text{seg } BC \cong \text{seg } QR$$

$$\angle ACB \cong \angle PRQ$$

$$\therefore \triangle ABC \cong \triangle PQR \dots\dots \boxed{} \text{ test}$$

$$\therefore \angle BAC \cong \boxed{} \dots\dots \text{corresponding angles of congruent triangles.}$$

$$\text{seg } AB \cong \boxed{\phantom{\text{seg}}}, \text{ and } \boxed{\phantom{\text{seg}}} \cong \text{seg } PR \left. \begin{array}{l} \text{corresponding} \\ \text{sides of congruent} \\ \text{triangles} \end{array} \right\}$$

(ii)

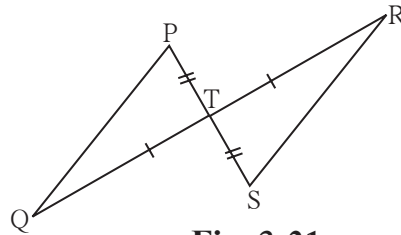


Fig. 3.21

From the information shown in the figure,,
In $\triangle PTQ$ and $\triangle STR$

$$\text{seg } PT \cong \text{seg } ST$$

$$\angle PTQ \cong \angle STR \dots\dots \text{vertically opposite angles}$$

$$\text{seg } TQ \cong \text{seg } TR$$

$$\therefore \triangle PTQ \cong \triangle STR \dots\dots \boxed{} \text{ test}$$

$$\therefore \angle TPQ \cong \boxed{} \text{ and } \boxed{} \cong \angle TRS \left. \begin{array}{l} \text{corresponding} \\ \text{angles of congruent} \\ \text{triangles.} \end{array} \right\}$$

$$\text{seg } PQ \cong \boxed{\phantom{\text{seg}}} \text{ corresponding sides of congruent triangles.}$$

3. From the information shown in the figure, state the test assuring the congruence of $\triangle ABC$ and $\triangle PQR$. Write the remaining congruent parts of the triangles.



Fig. 3.22

4. As shown in the following figure, in $\triangle LMN$ and $\triangle PNM$, $LM = PN$, $LN = PM$. Write the test which assures the congruence of the two triangles. Write their remaining congruent parts.

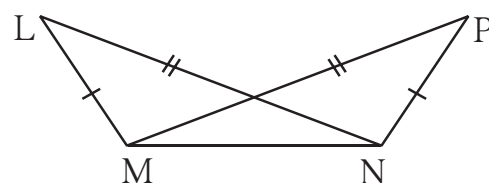


Fig. 3.23

5. In figure 3.24, $\text{seg } AB \cong \text{seg } CB$
and $\text{seg } AD \cong \text{seg } CD$.
Prove that
 $\triangle ABD \cong \triangle CBD$

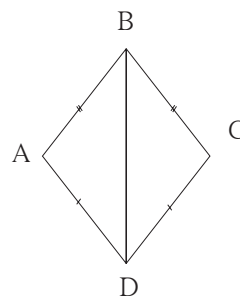


Fig. 3.24

Please note : corresponding sides of congruent triangles in short we write c.s.c.t. and corresponding angles of congruent triangles in short we write c.a.c.t.

6. In figure 3.25, $\angle P \cong \angle R$
 $\text{seg PQ} \cong \text{seg RQ}$
 Prove that,
 $\triangle PQT \cong \triangle RQS$

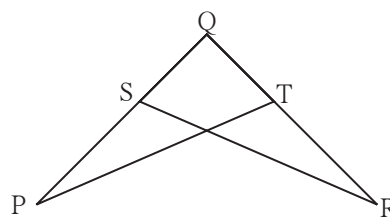


Fig. 3.25



Let's learn.

Isosceles triangle theorem

Theorem : If two sides of a triangle are congruent then the angles opposite to them are congruent.

Given : In $\triangle ABC$, side $AB \cong$ side AC

To prove : $\angle ABC \cong \angle ACB$

Construction : Draw the bisector of $\angle BAC$
 which intersects side BC at point D .

Proof : In $\triangle ABD$ and $\triangle ACD$
 $\text{seg AB} \cong \text{seg AC}$ given
 $\angle BAD \cong \angle CAD$ construction
 $\text{seg AD} \cong \text{seg AD}$ common side
 $\therefore \triangle ABD \cong \triangle ACD$
 $\therefore \angle ABD \cong$ (c.a.c.t.)
 $\therefore \angle ABC \cong \angle ACB$ $\because B - D - C$

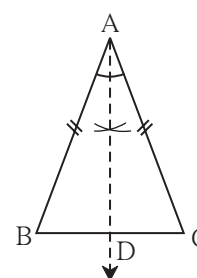


Fig. 3.26

Corollary : If all sides of a triangle are congruent then its all angles are congruent.
 (write the proof of this corollary.)

Converse of isosceles triangle theorem

Theorem : If two angles of a triangle are congruent then the sides opposite to them are congruent.

Given : In $\triangle PQR$, $\angle PQR \cong \angle PRQ$

To prove : Side $PQ \cong$ side PR

Construction : Draw the bisector of $\angle P$
 intersecting side QR at point M

Proof : In $\triangle PQM$ and $\triangle PRM$
 $\angle PQM \cong$ given
 $\angle QPM \cong \angle RPM$
 $\text{seg PM} \cong$ common side
 $\therefore \triangle PQM \cong \triangle PRM$ test
 $\therefore \text{seg PQ} \cong \text{seg PR}$ c.s.c.t.

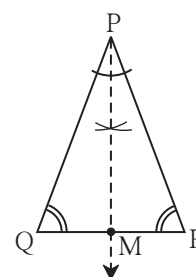


Fig. 3.27

Corollary: If three angles of a triangle are congruent then its three sides also are congruent.
(Write the proof of this corollary yourself.)

Both the above theorems are converses of each other also.

Similarly the corollaries of the theorems are converses of each other.



Use your brain power!

- (1) Can the theorem of isosceles triangle be proved doing a different construction ?
- (2) Can the theorem of isosceles triangle be proved without doing any construction ?



Let's learn.

Property of $30^\circ - 60^\circ - 90^\circ$ triangle

Activity I

Every student in the group should draw a right angled triangle, one of the angles measuring 30° . The choice of lengths of sides should be their own. Each one should measure the length of the hypotenuse and the length of the side opposite to 30° angle.

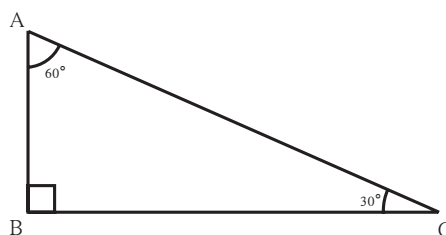


Fig. 3.28

One of the students in the group should fill in the following table.

Triangle Number	1	2	3	4
Length of the side opposite to 30° angle				
Length of the hypotenuse				

Did you notice any property of sides of right angled triangle with one of the angles measuring 30° ?

Activity II

The measures of angles of a set square in your compass box are 30° , 60° and 90° .
Verify the property of the sides of the set square.

Let us prove an important property revealed from these activities.

Given : In ΔABC
 $\angle B = 90^\circ, \angle C = 30^\circ, \angle A = 60^\circ$

A right-angled triangle ABC is shown. The right angle is at vertex B, indicated by a square symbol. Angle A is labeled as 60° and angle C is labeled as 30° .

Construction : Take a point D on the extended seg AB such that $AB = BD$. Draw seg DC.

 $\text{seg AB} \cong \text{seg DB}$
$$\angle ABC \cong \angle DBC \dots\dots\dots$$
$$\text{seg BC} \cong \text{seg BC} \dots\dots\dots$$
$$\therefore \triangle ABC \cong \triangle DBC \dots$$
$$\therefore \angle BAC \cong \angle BDC \dots\dots\dots (\text{c.a.c.t.})$$

In ΔABC , $\angle BAC = 60^\circ \therefore \angle BDC = 60^\circ$

$$\angle DAC = \angle ADC = \angle ACD = 60^\circ \dots \because \text{sum of angles of } \Delta \text{ ADC is } 180^\circ$$

$\therefore \Delta ADC$ is an equilateral triangle.

$\therefore AC = AD = DC$ corollary of converse of isosceles triangle theorem

But $AB = \frac{1}{2} AD$ construction

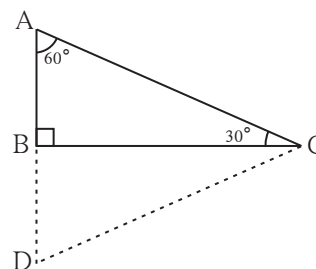
$$\therefore AB = \frac{1}{2} AC \dots\dots\dots \because AD = AC$$


Fig. 3.30

With the help of the Figure 3.29 above fill in the blanks and complete the proof of the following theorem.

Theorem : If the acute angles of a right angled triangle have measures 30° and 60° then the length of the side opposite to 60° angle is $\frac{\sqrt{3}}{2} \times \text{hypotenuse}$

Proof : In the above theorem we have proved $AB = \frac{1}{2} AC$

$$AB^2 + BC^2 = \boxed{} \dots\dots\dots \text{Pythagoras theorem}$$

$$\frac{1}{4} AC^2 + BC^2 = \boxed{}$$

$$\therefore \text{BC}^2 = \text{AC}^2 - \frac{1}{4} \text{AC}^2$$

$$\therefore \text{BC}^2 = \boxed{}$$

$$\therefore \text{BC} = \frac{\sqrt{3}}{2} \text{AC}$$

Activity : Complete the proof of the theorem.

Theorem : If measures of angles of a triangle are $45^\circ, 45^\circ, 90^\circ$ then the length of each side containing the right angle is $\frac{1}{\sqrt{2}} \times \text{hypotenuse}$.

Proof : In ΔABC , $\angle B = 90^\circ$ and $\angle A = \angle C = 45^\circ$

$$\therefore BC = AB$$

By Pythagoras theorem

$$AB^2 + BC^2 = \square$$

$$AB^2 + \square = AC^2 \dots \because (BC = AB)$$

$$\therefore 2AB^2 = \square$$

$$\therefore AB^2 = \square$$

$$\therefore AB = \frac{1}{\sqrt{2}} AC$$

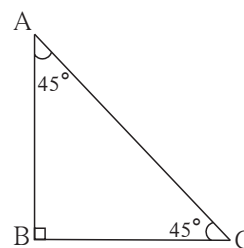


Fig. 3.31

This property is called $45^\circ - 45^\circ - 90^\circ$ theorem.



Remember this !

- (1) If the acute angles of a right angled triangle are $30^\circ, 60^\circ$ then the length of side opposite to 30° angle is half of hypotenuse and the length of side opposite to 60° angle is $\frac{\sqrt{3}}{2}$ hypotenuse . This property is called $30^\circ - 60^\circ - 90^\circ$ theorem.
- (2) If acute angles of a right angled triangle are $45^\circ, 45^\circ$ then the length of each side containing the right angle is $\frac{\text{hypotenuse}}{\sqrt{2}}$.

This property is called $45^\circ - 45^\circ - 90^\circ$ theorem.



Let's recall.

Median of a triangle

The segment joining a vertex and the mid-point of the side opposite to it is called a **Median** of the triangle.

In Figure 3.32, point D is the mid point of side BC.

$\therefore$ seg AD is a median of ΔABC .

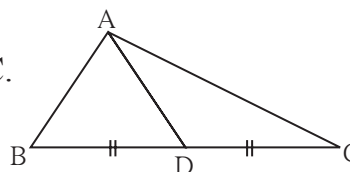


Fig. 3.32

Activity I : Draw a triangle ABC. Draw medians AD, BE and CF of the triangle. Let their point of concurrence be G, which is called the centroid of the triangle. Compare the lengths of AG and GD with a divider. Verify that the length of AG is twice the length of GD. Similarly, verify that the length of BG is twice the length of GE and the length of CG is twice the length of GF. Hence note the following property of medians of a triangle.

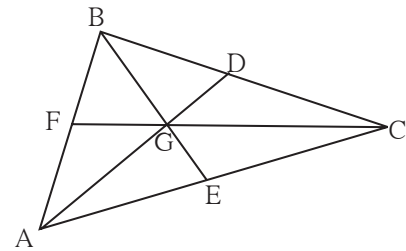


Fig. 3.33

The point of concurrence of medians of a triangle divides each median in the ratio 2 : 1.

Activity II : Draw a triangle ABC on a card board. Draw its medians and denote their point of concurrence as G. Cut out the triangle.

Now take a pencil. Try to balance the triangle on the flat tip of the pencil. The triangle is balanced only when the point G is on the flat tip of the pencil.

This activity shows an important property of the **centroid** (point of concurrence of the medians) of the triangle.

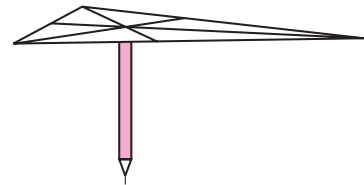


Fig. 3.34



Let's learn.

Property of median drawn on the hypotenuse of right triangle

Activity : In the figure 3.35, $\triangle ABC$ is a right angled triangle. seg BD is the median on hypotenuse.

Measure the lengths of the following segments.

AD = DC = BD =

From the measurements verify that $BD = \frac{1}{2} AC$.

Now let us prove the property, the length of the median is half the length of the hypotenuse.

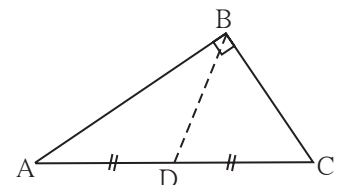


Fig. 3.35

Theorem : In a right angled triangle, the length of the median of the hypotenuse is half the length of the hypotenuse.

Given : In ΔABC , $\angle B = 90^\circ$, seg BD is the median.

To prove : $BD = \frac{1}{2} AC$

Construction : Take point E on the ray BD such that B - D - E
and $l(\text{BD}) = l(\text{DE})$. Draw seg EC.

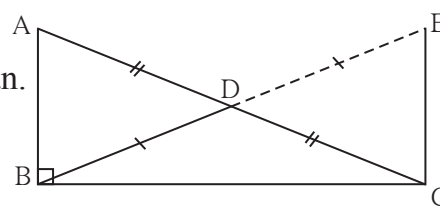


Fig. 3.36

Proof : (Main steps are given. Write the steps in between with reasons and complete the proof.)

$$\triangle ADB \cong \triangle CDE \text{ by S-A-S test}$$

line AB $\parallel$ line ECby test of alternate angles

$$\triangle ABC \cong \triangle ECB \text{ by S-A-S test}$$

$$BD = \frac{1}{2} AC$$



Remember this !

In a right angled triangle, the length of the median on its hypotenuse is half the length of the hypotenuse.

Practice set 3.3

1. Find the values of x and y using the information shown in figure 3.37.
Find the measure of $\angle ABD$ and $m\angle ACD$.

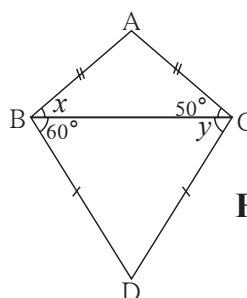


Fig. 3.37

- The length of hypotenuse of a right angled triangle is 15. Find the length of median of its hypotenuse.
- In $\triangle PQR$, $\angle Q = 90^\circ$, $PQ = 12$, $QR = 5$ and QS is a median. Find $l(QS)$.
- In figure 3.38, point G is the point of concurrence of the medians of $\triangle PQR$. If $GT = 2.5$, find the lengths of PG and PT .

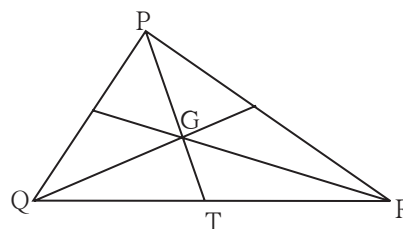


Fig. 3.38



Let's recall.

Activity : Draw a segment AB of convenient length. Label its mid-point as M. Draw a line l passing through the point M and perpendicular to seg AB.

Did you notice that the line l is the perpendicular bisector of seg AB ?

Now take a point P anywhere on line l . Compare the distance PA and PB with a divider. What did you find ? You should have noticed that $PA = PB$. This observation shows that any point on the perpendicular bisector of a segment is equidistant from its end points.

Now with the help of a compass take any two points like C and D, which are equidistant from A and B. Did all such points lie on the line l ? What did you notice from the observation ? Any point equidistant from the end points of a segment lies on the perpendicular bisector of the segment.

These two properties are two parts of the perpendicular bisector theorem. Let us now prove them.

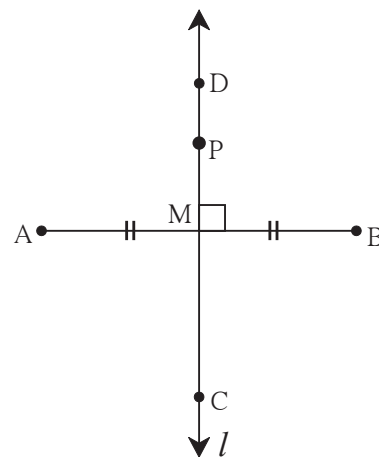


Fig. 3.39



Let's learn.

Perpendicular bisector theorem

Part I : Every point on the perpendicular bisector of a segment is equidistant from the end points of the segment.

Given : line l is the perpendicular bisector of seg AB at point M.

Point P is any point on l ,

To prove: $PA = PB$

Construction : Draw seg AP and seg BP.

Proof : In $\triangle PMA$ and $\triangle PMB$

seg PM $\cong$ seg PM common side

$\angle PMA \cong \angle PMB$ each is a right angle

seg AM $\cong$ seg BMgiven

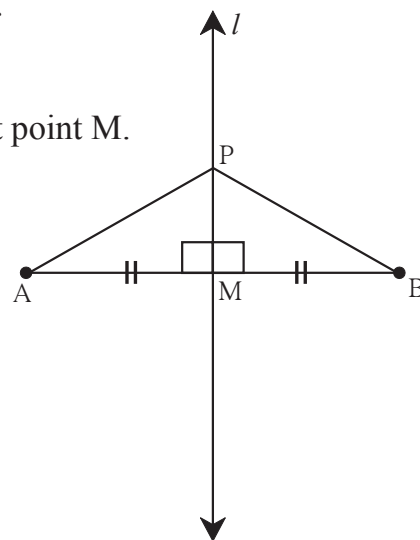


Fig. 3.40

$\therefore \triangle PMA \cong \triangle PMB$ S-A-S test

$\therefore \text{seg } PA \cong \text{seg } PB$ c.s.c.t.

$\therefore l(PA) = l(PB)$

Hence every point on the perpendicular bisector of a segment is equidistant from the end points of the segment.

Part II : Any point equidistant from the end points of a segment lies on the perpendicular bisector of the segment.

Given : Point P is any point equidistant from the end points of seg AB. That is, $PA = PB$.

To prove: Point P is on the perpendicular bisector of seg AB.

Construction : Take mid-point M of seg AB and draw line PM.

Proof : In $\triangle PAM$ and $\triangle PBM$

$\text{seg } PA \cong \text{seg } PB$

$\text{seg } AM \cong \text{seg } BM$

$\text{seg } PM \cong$ common side

$\therefore \triangle PAM \cong \triangle PBM$ test.

$\therefore \angle PMA \cong \angle PMB$c.a.c.t.

But $\angle PMA +$ $= 180^\circ$

$\angle PMA + \angle PMB = 180^\circ$ ($\because \angle PMB = \angle PMA$)

$2 \angle PMA =$

$\therefore \angle PMA = 90^\circ$

$\therefore \text{seg } PM \perp \text{seg } AB$ (1)

But Point M is the midpoint of seg AB.construction (2)

$\therefore$ line PM is the perpendicular bisector of seg AB. So point P is on the perpendicular bisector of seg AB.

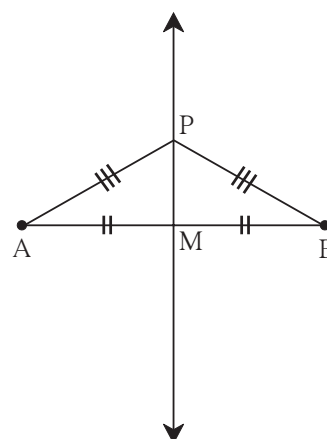


Fig. 3.41

Angle bisector theorem

Part I : Every point on the bisector of an angle is equidistant from the sides of the angle.

Given : Ray QS is the bisector of $\angle PQR$.
Point A is any point on ray QS
 $\text{seg } AB \perp \text{ray } QP$ $\text{seg } AC \perp \text{ray } QR$

To prove : $\text{seg } AB \cong \text{seg } AC$

Proof : Write the proof using test of congruence of triangles.

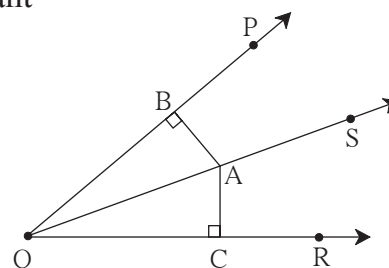


Fig. 3.42



Part II : Any point equidistant from sides of an angle is on the bisector of the angle.

Given : A is a point in the interior of $\angle PQR$.
 $\text{seg } AC \perp \text{ray } QR$ $\text{seg } AB \perp \text{ray } QP$
 and $AB = AC$

To prove : Ray QA is the bisector of $\angle PQR$.
 That is $\angle BQA = \angle CQA$

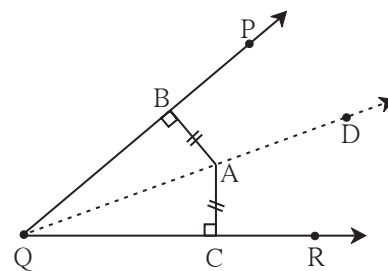


Fig. 3.43

Proof : Write the proof using proper test of congruence of triangles.



Activity

As shown in the figure, draw $\triangle XYZ$
 such that $XZ > \text{side } XY$
 Find which of $\angle Z$ and $\angle Y$ is greater.

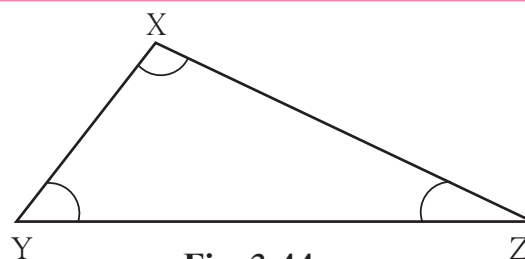


Fig. 3.44



Properties of inequalities of sides and angles of a triangle

Theorem : If two sides of a triangle are unequal, then the angle opposite to the greater side is greater than angle opposite to the smaller side.

Given : In $\triangle XYZ$, side $XZ > \text{side } XY$

To prove : $\angle XYZ > \angle XZY$

Construction : Take point P on side XZ such that
 $XY = XP$, Draw seg YP.

Proof : In $\triangle XYP$

$XY = XP$ construction

$\therefore \angle XYP = \angle XPY$isosceles triangle theorem(I)

$\angle XPY$ is an exterior angle of $\triangle YPZ$.

$\therefore \angle XPY > \angle PZY$ exterior angle theorem

$\therefore \angle XYP > \angle PZY$ from (I)

$\therefore \angle XYP + \angle PYZ > \angle PZY$ If $a > b$ and $c > 0$ then $a + c > b$

$\therefore \angle XYZ > \angle PZY$, that is $\angle XYZ > \angle XZY$

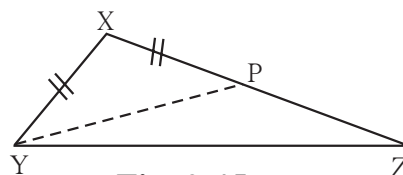


Fig. 3.45

Theorem : If two angles of a triangle are unequal then the side opposite to the greater angle is greater than the side opposite to smaller angle.

The theorem can be proved by indirect proof. Complete the following proof by filling in the blanks.

Given : In $\triangle ABC$, $\angle B > \angle C$

To prove : $AC > AB$

Proof : There are only three possibilities regarding the lengths of side AB and side AC of ΔABC .

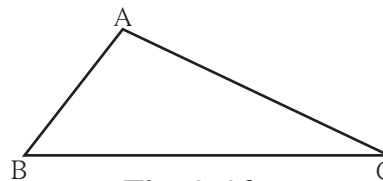


Fig. 3.46

(i) $AC < AB$

(ii)

(iii)

(i) Let us assume that $AC < AB$.

If two sides of a triangle are unequal then the angle opposite to greater side is .

$$\therefore \angle C > \boxed{}$$

But $\angle C < \angle B$ (given)

This creates a contradiction.

$\therefore \square < \square$ is wrong.

(ii) If $AC = AB$

then $\angle B = \angle C$

But $\square > \square$ (given)

This also creates a contradiction.

$\therefore \square = \square$ is wrong

$\therefore AC > AB$ is the only remaining possibility.

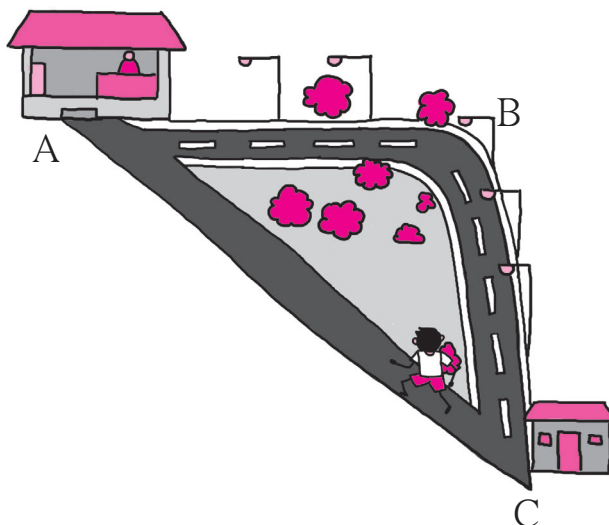
$$\therefore AC > AB$$


Let's recall.

As shown in the adjacent picture, there is a shop at A. Sameer was standing at C. To reach the shop, he choose the way $C \rightarrow A$ instead of $C \rightarrow B \rightarrow A$, because he knew that the way $C \rightarrow A$ was shorter than the way $C \rightarrow B \rightarrow A$. So which property of a triangle had he realised ?

The sum of two sides of a triangle is greater than its third side.

Let us now prove the property.



Theorem : The sum of any two sides of a triangle is greater than the third side.

Given : $\triangle ABC$ is any triangle.

To prove : $AB + AC > BC$
 $AB + BC > AC$
 $AC + BC > AB$

Construction : Take a point D on ray BA such that $AD = AC$.

Proof : In $\triangle ACD$, $AC = AD$ construction

$\therefore \angle ACD = \angle ADC$ c.a.c.t.

$\therefore \angle ACD + \angle ACB > \angle ADC$

$\therefore \angle BCD > \angle ADC$

$\therefore$ side $BD >$ side BC the side opposite to greater angle is greater

$\therefore BA + AD > BC$ $\because BD = BA + AD$

$BA + AC > BC$ $\because AD = AC$

Similarly we can prove that $AB + BC > AC$

and $BC + AC > AB$.

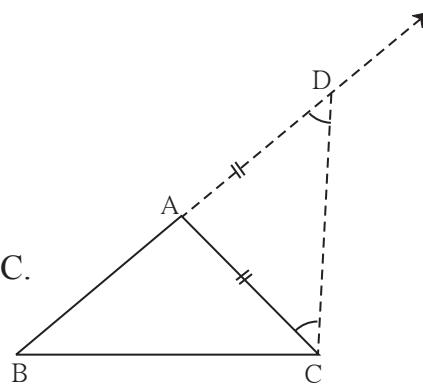


Fig. 3.47

Practice set 3.4

1. In figure 3.48, point A is on the bisector of $\angle XYZ$.

If $AX = 2$ cm then find AZ .

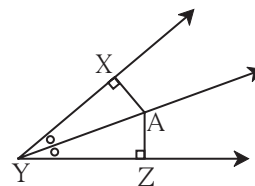


Fig. 3.48

2.

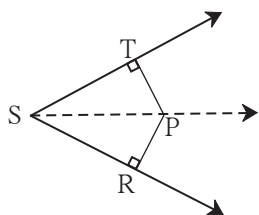


Fig. 3.49

In figure 3.49, $\angle RST = 56^\circ$, seg $PT \perp$ ray ST ,

seg $PR \perp$ ray SR and seg $PR \cong$ seg PT

Find the measure of $\angle RSP$.

State the reason for your answer.

3. In $\triangle PQR$, $PQ = 10$ cm, $QR = 12$ cm, $PR = 8$ cm. Find out the greatest and the smallest angle of the triangle.
4. In $\triangle FAN$, $\angle F = 80^\circ$, $\angle A = 40^\circ$. Find out the greatest and the smallest side of the triangle. State the reason.
5. Prove that an equilateral triangle is equiangular.

6. Prove that, if the bisector of $\angle BAC$ of $\triangle ABC$ is perpendicular to side BC , then $\triangle ABC$ is an isosceles triangle.

7. In figure 3.50, if $\text{seg } PR \cong \text{seg } PQ$, show that $\text{seg } PS > \text{seg } PQ$.

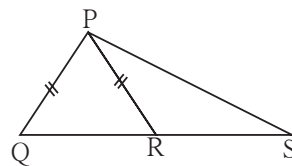


Fig. 3.50

8. In figure 3.51, in $\triangle ABC$, $\text{seg } AD$ and $\text{seg } BE$ are altitudes and $AE = BD$.

Prove that $\text{seg } AD \cong \text{seg } BE$

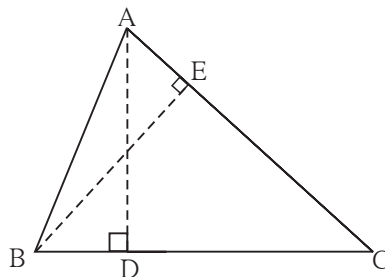


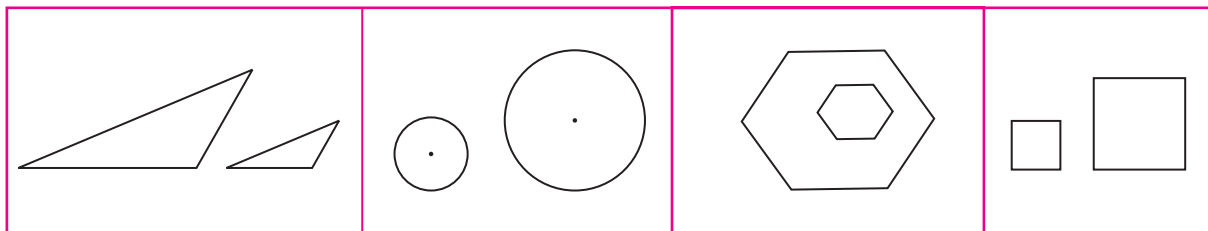
Fig. 3.51



Let's learn.

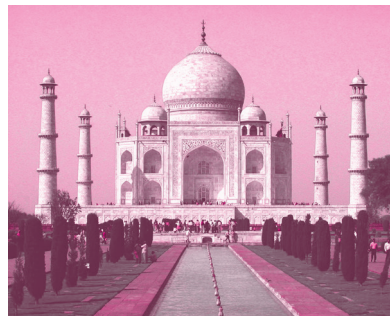
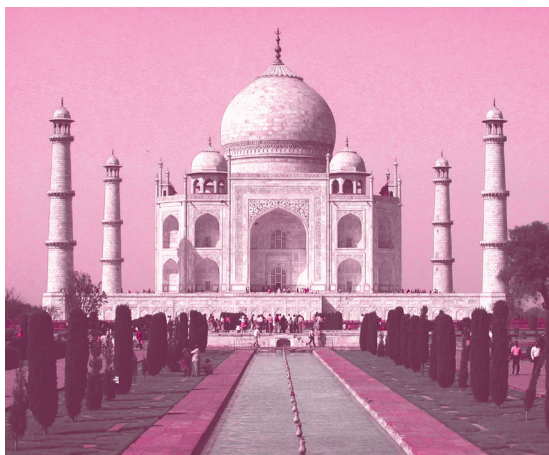
Similar triangles

Observe the following figures.



The pairs of figures shown in each part have the same shape but their sizes are different. It means that they are not congruent.

Such like looking figures are called similar figures.



We find similarity in a photo and its enlargement, also we find similarity between a road-map and the roads.



Take a photograph on a mobile or a computer. Recall what you do to reduce it or to enlarge it. Also recall what you do to see a part of the photograph in detail.

Activity : On a card-sheet, draw a triangle of sides 4 cm, 3 cm and 2 cm. Cut it out. Make 13 more copies of the triangle and cut them out from the card sheet.

Note that all these triangular pieces are congruent. Arrange them as shown in the following figure and make three triangles out of them.

ΔABC and ΔDEF are similar in the correspondence $ABC \leftrightarrow DEF$.



Let's learn.

Similarity of triangles

In $\triangle ABC$ and $\triangle PQR$, If (i) $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$ and

(ii) $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$; then $\triangle ABC$ and $\triangle PQR$ are called similar triangles.

' $\triangle ABC$ and $\triangle PQR$ are similar' is written as ' $\triangle ABC \sim \triangle PQR$ '.

Let us learn the relation between the corresponding angles and corresponding sides of similar triangles through an activity.

Activity : Draw a triangle $\triangle A_1B_1C_1$ on a card-sheet and cut it out. Measure $\angle A_1, \angle B_1, \angle C_1$.

Draw two more triangles $\triangle A_2B_2C_2$ and $\triangle A_3B_3C_3$ such that

$\angle A_1 = \angle A_2 = \angle A_3$, $\angle B_1 = \angle B_2 = \angle B_3$, $\angle C_1 = \angle C_2 = \angle C_3$

and $B_1C_1 > B_2C_2 > B_3C_3$. Now cut these two triangles also. Measure the lengths of the three triangles. Arrange the triangles in two ways as shown in the figure.

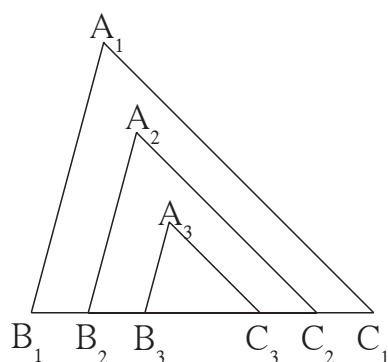


Fig. 3.55

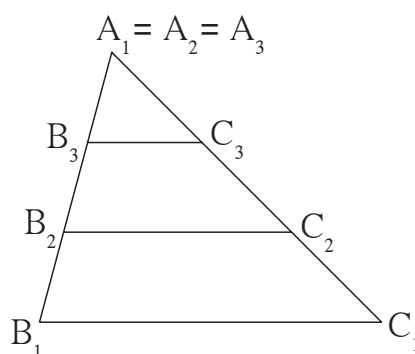


Fig. 3.56

Check the ratios $\frac{A_1B_1}{A_2B_2}$, $\frac{B_1C_1}{B_2C_2}$, $\frac{A_1C_1}{A_2C_2}$. You will notice that the ratios are equal.

Similarly, see whether the ratios $\frac{A_1C_1}{A_3C_3}$, $\frac{B_1C_1}{B_3C_3}$, $\frac{A_1B_1}{A_3B_3}$ are equal.

From this activity note that, when corresponding angles of two triangles are equal, the ratios of their corresponding sides are also equal. That is, their corresponding sides are in the same proportion.

We have seen that, in $\triangle ABC$ and $\triangle PQR$ if

(i) $\angle A = \angle P$, $\angle B = \angle Q$, $\angle C = \angle R$, then (ii) $\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$

This means, if corresponding angles of two triangles are equal then the corresponding sides are in the same proportion.

This rule can be proved elaborately. We shall use it to solve problems.





Remember this !

- If corresponding angles of two triangles are equal then the two triangles are similar.
- If two triangles are similar then their corresponding sides are in proportion and corresponding angles are congruent.

Ex. Some information is shown in $\triangle ABC$ and $\triangle PQR$ in figure 3.57. Observe it. Hence find the lengths of side AC and PQ.

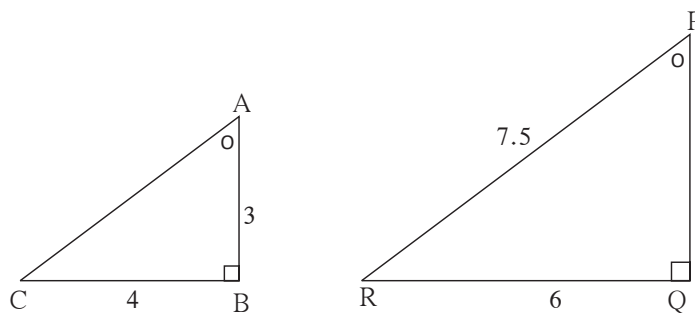


Fig. 3.57

Solution : The sum of all angles of a triangle is 180° .

It is given that,

$$\angle A = \angle P \text{ and } \angle B = \angle Q \quad \therefore \angle C = \angle R$$

$\therefore \triangle ABC$ and $\triangle PQR$ are equiangular triangles.

$\therefore$ their sides are proportional.

$$\therefore \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$

$$\therefore \frac{3}{PQ} = \frac{4}{6} = \frac{AC}{7.5}$$

$$\therefore 4 \times PQ = 18$$

$$\therefore PQ = \frac{18}{4} = 4.5$$

$$\text{Similarly } 6 \times AC = 7.5 \times 4$$

$$\therefore AC = \frac{7.5 \times 4}{6} = \frac{30}{6} = 5$$

Practice set 3.5

1. If $\triangle XYZ \sim \triangle LMN$, write the corresponding angles of the two triangles and also write the ratios of corresponding sides.
2. In $\triangle XYZ$, $XY = 4$ cm, $YZ = 6$ cm, $XZ = 5$ cm, If $\triangle XYZ \sim \triangle PQR$ and $PQ = 8$ cm then find the lengths of remaining sides of $\triangle PQR$.
3. Draw a sketch of a pair of similar triangles. Label them. Show their corresponding angles by the same signs. Show the lengths of corresponding sides by numbers in proportion.



Let's recall.

While preparing a map of a locality, you have to show the distances between different spots on roads with a proper scale. For example, 1 cm = 100 m, 1 cm = 50 m etc. Did you think of the properties of triangle ? Keep in mind that side opposite to greater angle is greater.

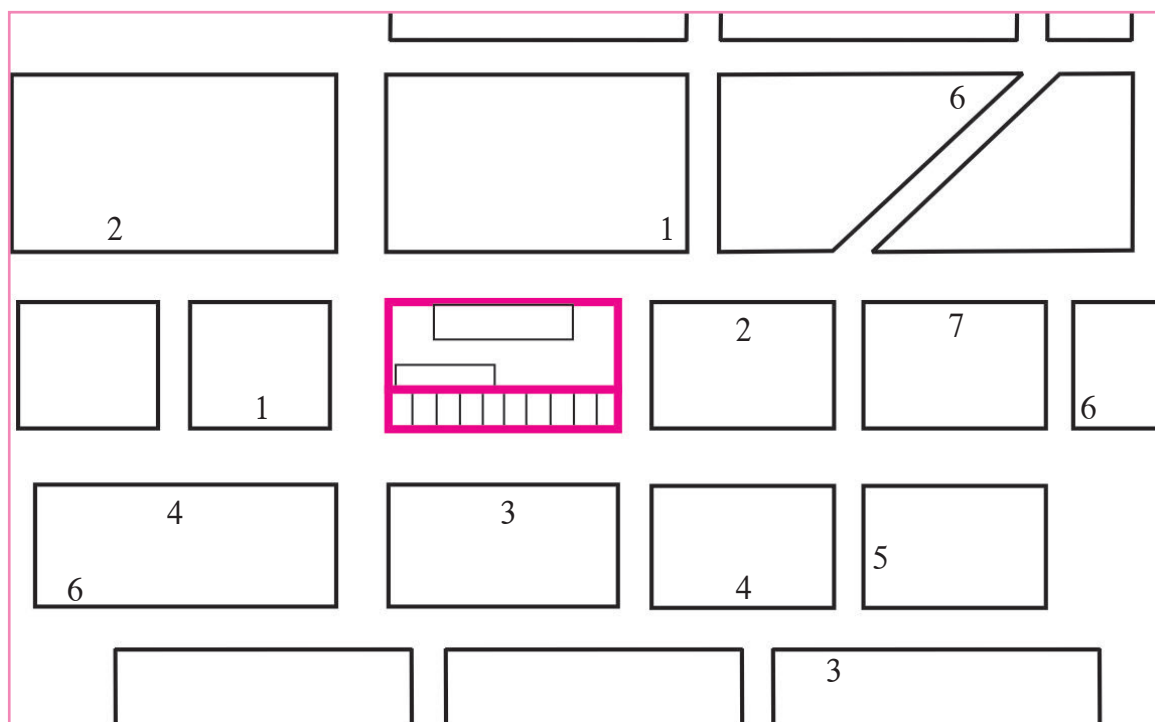
Project :

Prepare a map of road surrounding your school or home, upto a distance of about 500 metre.

How will you measure the distance between two spots on a road ?

While walking, count how many steps cover a distance of about two metre. Suppose, your three steps cover a distance of 2 metre. Considering this proportion 90 steps means 60 metre. In this way you can judge the distances between different spots on roads and also the lengths of roads. You have to judge the measures of angles also where two roads meet each other. Choosing a proper scale for lengths of roads, prepare a map. Try to show shops, buildings, bus stops, rickshaw stand etc. in the map.

A sample map with legend is given below.



Legend: 1. Book store 2. Bus stop 3. Stationery shop 4. Bank
5. Medical store 6. Restaurant 7. Cycle shop

Problem set 3

1. Choose the correct alternative answer for the following questions.
 - (i) If two sides of a triangle are 5 cm and 1.5 cm, the length of its third side cannot be
 (A) 3.7 cm (B) 4.1 cm (C) 3.8 cm (D) 3.4 cm
 - (ii) In $\triangle PQR$, If $\angle R > \angle Q$ then
 (A) $QR > PR$ (B) $PQ > PR$ (C) $PQ < PR$ (D) $QR < PR$
 - (iii) In $\triangle TPQ$, $\angle T = 65^\circ$, $\angle P = 95^\circ$ which of the following is a true statement ?
 (A) $PQ < TP$ (B) $PQ < TQ$ (C) $TQ < TP < PQ$ (D) $PQ < TP < TQ$
2. $\triangle ABC$ is isosceles in which $AB = AC$. Seg BD and seg CE are medians. Show that $BD = CE$.

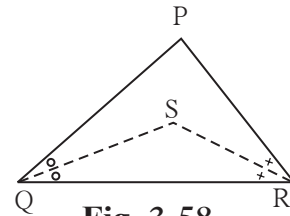


Fig. 3.58

4. In figure 3.59, point D and E are on side BC of $\triangle ABC$, such that $BD = CE$ and $AD = AE$. Show that $\triangle ABD \cong \triangle ACE$.

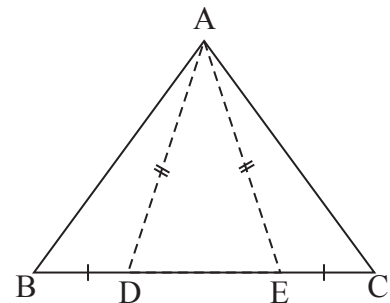


Fig. 3.59

5. In figure 3.60, point S is any point on side QR of $\triangle PQR$. Prove that : $PQ + QR + RP > 2PS$

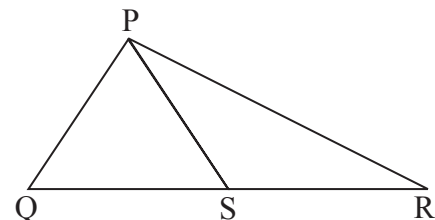


Fig. 3.60

6. In figure 3.61, bisector of $\angle BAC$ intersects side BC at point D .
Prove that $AB > BD$

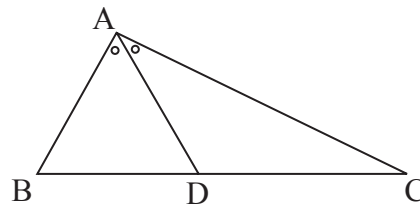


Fig. 3.61

7.

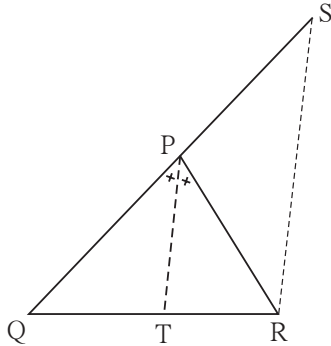


Fig. 3.62

In figure 3.62, seg PT is the bisector of $\angle QPR$.
A line parallel to seg PT and passing through R intersects ray QP at point S . Prove that $PS = PR$.

8. In figure 3.63, seg $AD \perp$ seg BC .
seg AE is the bisector of $\angle CAB$ and
 $C - E - D$.
Prove that
$$\angle DAE = \frac{1}{2} (\angle C - \angle B)$$

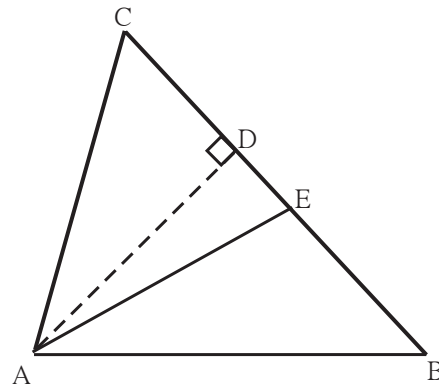


Fig. 3.63



Use your brain power!

We have learnt that if two triangles are equiangular then their sides are in proportion. What do you think if two quadrilaterals are equiangular? Are their sides in proportion? Draw different figures and verify.

Verify the same for other polygons.

