7 Co-ordinate Geometry

24NVP1

Let's study.

- **Axis, Origin, Quadrant**
- **Co-ordinates of a point in a plane.**
- **To plot a point.**
- **Line parallel to X-axis.**
- **Line parallel to Y-axis.**
- **Equation of a line.**

Chintu and his friends were playing cricket on the ground in front of a big building, when a visitor arrived.

Visitor : Hey Chintu, Dattabhau lives here, doesn't he ?

Chintu : Yes, on the second floor. See that window ? Thats his flat.

Visitor : But there are five windows on the second floor. It could be any of them !

Chintu : His window is the third one from the left, on the second floor.

Chintu's description of the location of Dattabhau's flat is in fact, based on the most basic concept in Co-ordinate Geometry.

It did not suffice to give only the floor number to locate the house. Its serial number from the left or from the right also needed to be given. That is two numbers had to be given in a specific sequence. Two **ordinal numbers** namely, **second** from the ground and **third** from the left had to be used.

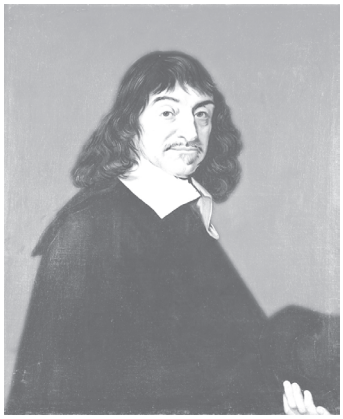
Let's learn.

Axis, origin, quadrants

We could give the location of Dattabhau's house using two ordinal numbers. Similarly, the location of a point can be fully described using its distances from two mutually perpendicular lines.

To locate a point in a plane, a horizontal number line is drawn in the plane. This number line is called the X-axis.

Rene Descartes (1596-1650)



Rene Descartes, a French mathematician of the 17th Century, proposed the co-ordinate system to describe the position of a point in a plane accurately. It is called the Cartesian co-ordinate system. Obviously the word Cartesian is derived from his name. He brought about a revolution in the field of mathematics by establishing the relationship between Algebra and Geometry.

The Cartesian co-ordinate system is the foundation of Analytical Geometry. La Geometric was Descartes' first book on mathematics. In it, he used algebra for the study of geometry and proposed that a point in a plane can be represented by an ordered pair of real numbers. This ordered pair is the 'Cartesian Co-ordinates' of a point.

Co-ordinate geometry has used in a variety of fields such as Physics, Engineering, Nautical Science, Siesmology and Art. It plays an important role in the development of technology in Geogebra. We see the inter-relationship between Algebra and Geometry quite clearly in the software Geogebra;the very name being a combination of the words 'Geometry' and 'Algebra'.

Another number line intersecting the X-axis at point marked O and perpendicular to the X-axis, is the Y-axis. Generally, the number O is represented by the same point on both the number lines. This point is called the origin and is shown by the letter O.

On the X-axis, positive numbers are shown on the right of O and negative numbers on the left.

On the Y-axis, positive numbers are shown above O and negative numbers below it.

The X and Y axes divide the plane into four parts, each of which is called a **Quadrant.** As shown in the figure, the quadrants are numbered in the anti-clocksise direction.

The points on the axes are not included in the quadrants.

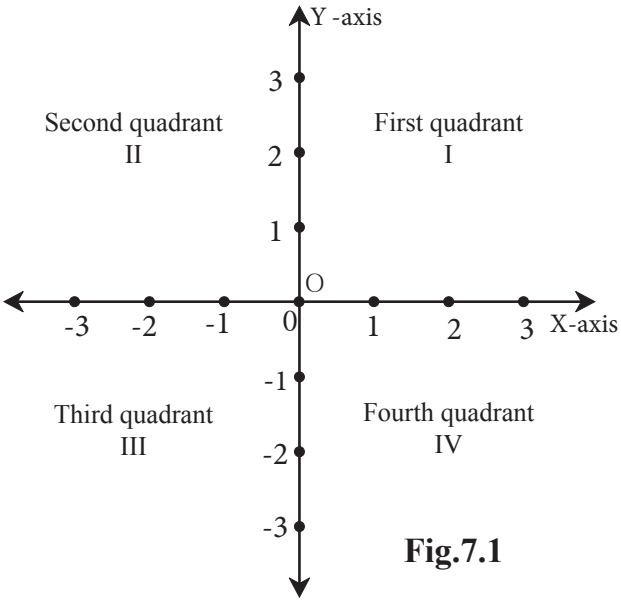


Fig.7.1

The Co-ordinates of a point in a plane

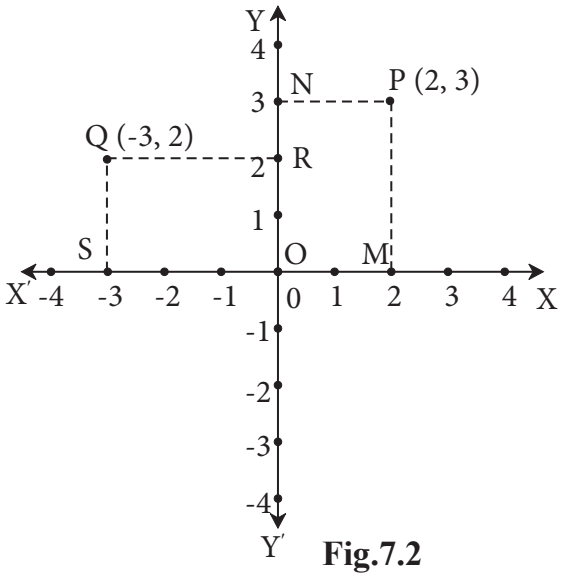


Fig.7.2

The point P is shown in the plane determined by the X-axis and the Y-axis. Its position can be determined by its distance from the two axes. To find these distances, we draw seg PM ⊥ X-axis and seg PN ⊥ Y-axis.

Co-ordinate of point M on X-axis is 2 and co-ordinate of point N on Y-axis is 3.

Therefore x co-ordinate of point P is 2 and y co-ordinate of point P is 3.

The convention for describing the position of a point is to mention x co-ordinate first. According to this convention the order of co-ordinates of point P is decided as 2, 3. The position of the point P in brief, is described by the pair (2, 3)

The order of the numbers in the pair (2, 3) is important. Such a pair of numbers is called an ordered pair.

To describe the position of point Q, we draw seg QS ⊥ X-axis and seg QR ⊥ Y-axis. The co-ordinate of point Q on the X-axis is –3 and the co-ordinate on the Y-axis is 2. Hence the co-ordinates of point Q are (–3, 2).

Ex. Write the co-ordinates of points E, F, G, T in the figure alongside.

Solution :

- The co-ordinates of point E are (2, 1)
- The co-ordinates of point F are (–3, 3)
- The co-ordinates of point G are (–4, –2).
- The co-ordinates of point T are (3, –1)

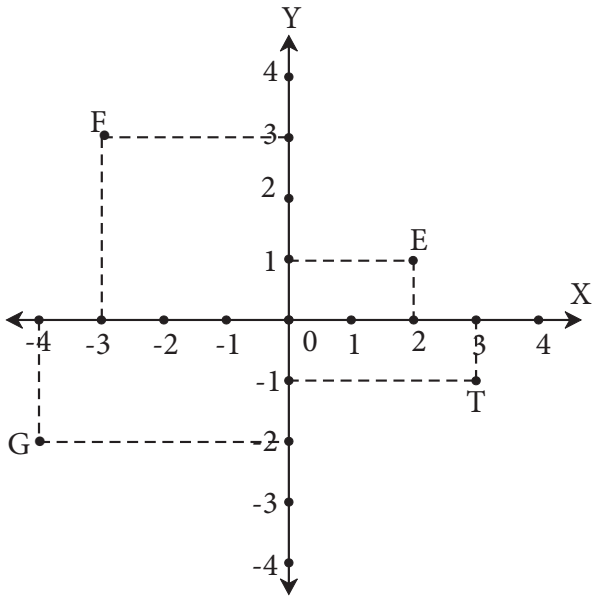


Fig.7.3

Let's learn.

Co-ordinates of points on the axes

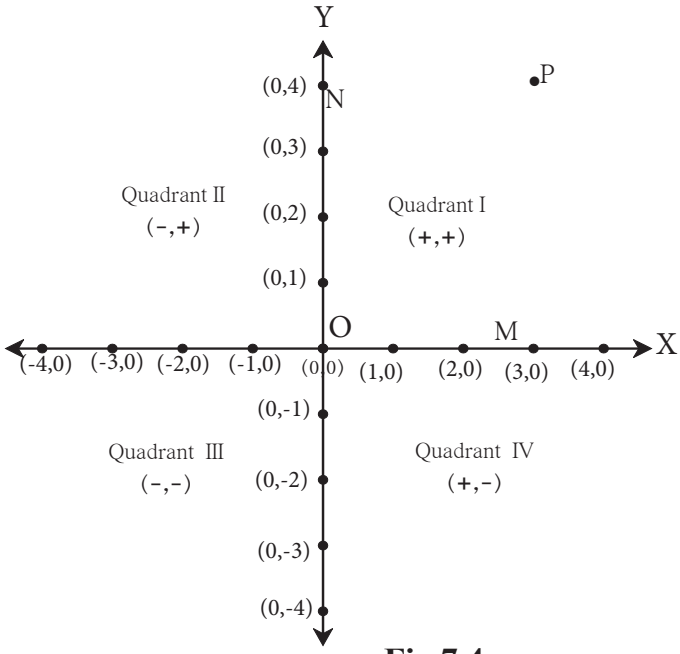


Fig.7.4

The x co-ordinate of point M is its distance from the Y-axis. The distance of point M from the X-axis is zero. Hence, the y co-ordinate of M is 0.

Thus, the co-ordinates of point M on the X-axis are (3,0).

The y co-ordinate of point N on the Y-axis is 4 units from the X-axis because N is at a distance of 4. Its x co-ordinate is 0 because its distance from the Y-axis is zero.

Hence, the co-ordinates of point N on the Y-axis are (0, 4).

Now the origin 'O' is on X-axis as well as on Y-axis. Hence, its distance from X-axis and Y-axis is zero. Therefore, the co-ordinates of O are (0, 0).

One and only one pair of co-ordinates (ordered pair) is associated with every point in a plane.

Let's Remember

- The y co-ordinate of every point on the X-axis is zero.
- The x co-ordinate of every point on the Y-axis is zero.
- The coordinates of the origin are (0, 0).

Ex. In which quadrant or on which axis are the points given below ?
A(5,7), B(–6,4), C(4,–7), D(–8,–9), P(–3,0), Q(0,8)

Solution : The x co-ordinate of A (5, 7) is positive and its y co-ordinate is positive..
∴ point A is in the first quadrant.
The x co-ordinate of B (–6, 4) is negative and y co-ordinate is positive.
∴ point B is in the second quadrant.
The x co-ordinate of C (4, –7) is positive and y co-ordinate is negative.
∴ point C is in the fourth quadrant.
The x co-ordinate of D (–8, –7) is negative and y co-ordinate is negative.
∴ point D is in the third quadrant.

The y co-ordinate of P(-3,0) is zero $\therefore$ point P is on the X-axis.

The x co-ordinate of Q (0,8) is zero $\therefore$ point Q is on the Y-axis.

Activity As shown in fig. 7.5, ask girls to sit in lines so as to form the X-axis and Y-axis.

- Ask some boys to sit at the positions marked by the coloured dots in the four quadrants.
- Now, call the students turn by turn using the initial letter of each student's name. As his or her initial is called, the student stands and gives his or her own co-ordinates. For example Rajendra (2, 2) and Kirti (-1, 0)
- Even as they have fun during this field activity, the students will learn how to state the position of a point in a plane.

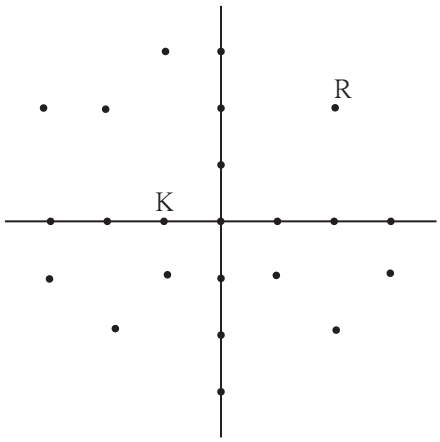


Fig. 7.5

Let's learn.

To plot the points of given co-ordinates

Suppose we have to plot the points P (4,3) and Q (-2,2)

Steps for plotting the points

(i) Draw X-axis and Y-axis on the plane. Show the origin.

(ii) To find the point P (4,3), draw a line parallel to the Y-axis through the point on X axis which represents the number 4.

Through the point on Y-axis which represents the number 3 draw a line parallel to the X-axis .

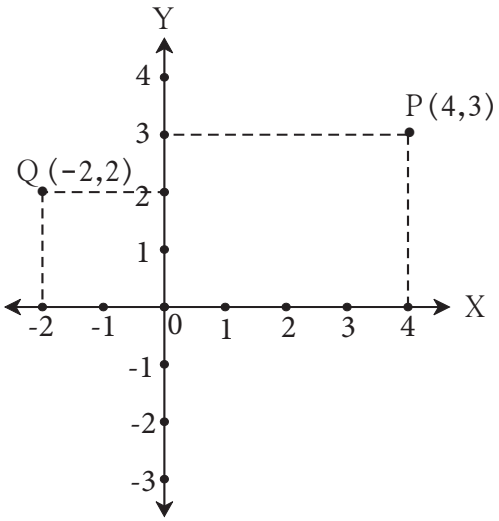


Fig. 7.6

(iii) The point of intersection of these two lines parallel to the Y and X-axis respectively, is the point P (4,3). In which quadrant does this point lie ?

(iv) In the same way, plot the point Q (–2, 2) . Is this point in the second quadrant ? Using the same method, plot the points R(–3, –4), S(3, –1)

Ex. In which quadrants or on which axis are the points given below ?

(i) (5, 3) (ii) (–2, 4) (iii) (2, –5) (iv) (0, 4)
(v) (–3, 0) (vi) (–2, 2.5) (vii) (5, 3.5) (viii) (–3.5, 1.5)
(ix) (0, –4) (x) (2, –4)

Solution :

	co-ordinates	Quadrant / axis
(i)	(5,3)	Quadrant I
(ii)	(–2,4)	Quadrant II
(iii)	(2,–5)	Quadrant IV
(iv)	(0,4)	Y–axis
(v)	(–3,0)	X–axis

	co-ordinates	Quadrant / axis
(vi)	(–2, –2.5)	Quadrant III
(vii)	(5,3.5)	Quadrant I
(viii)	(–3.5,1.5)	Quadrant II
(ix)	(0, –4)	Y–axis
(x)	(2,–4)	Quadrant IV

Practice set 7.1

1. State in which quadrant or on which axis do the following points lie.

- A(–3, 2),
- B(–5, –2),
- K(3.5, 1.5),
- D(2, 10),
- E(37, 35),
- F(15, –18),
- G(3, –7),
- H(0, –5),
- M(12, 0),
- N(0, 9),
- P(0, 2.5),
- Q(–7, –3)

2. In which quadrant are the following points ?

(i) whose both co-ordinates are positive.

(ii) whose both co-ordinates are negative.

(iii) whose x co-ordinate is positive, and the y co-ordinate is negative.

(iv) whose x co-ordinate is negative and y co-ordinate is positive.

3. Draw the co-ordinate system on a plane and plot the following points.

L(–2, 4), M(5, 6), N(–3, –4), P(2, –3), Q(6, –5), S(7, 0), T(0, –5)

Let's learn.

Lines parallel to the X-axis

- On a graph paper, plot the following points
  A (5, 4), B (2, 4), C (–2, 4), D (–4, 4), E (0, 4), F (3, 4)
- Observe the co-ordinates of the given points.
- Did you notice that the y co-ordinates of all the points are equal ?
- All the points are collinear.
- To which axis is this line parallel ?
- The y co-ordinate of every point on the line DA is 4. It is constant. Therefore the line DA is described by the equation $y = 4$. If the y co-ordinate of any point is 4, will be on the line DA.

  The equation of the line parallel to the X axis at a distance of 4 units from the X-axis is $y = 4$.

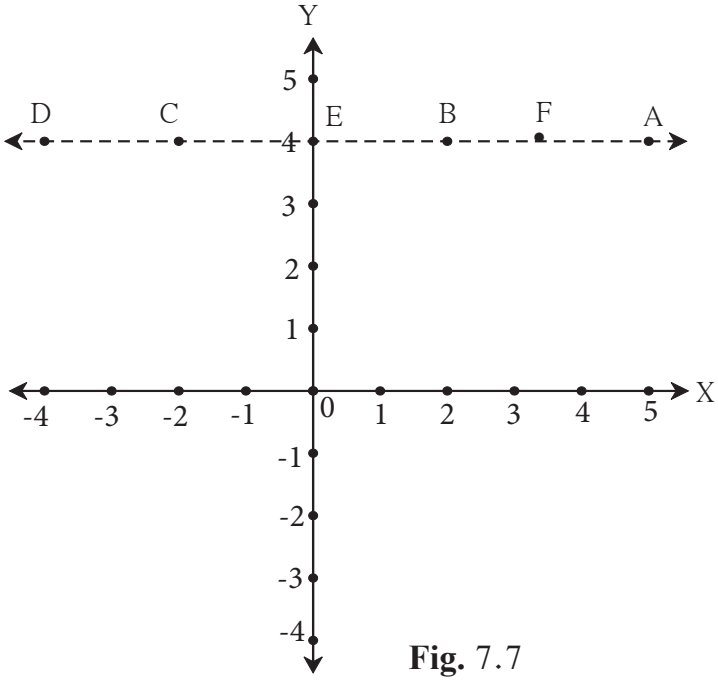


Fig. 7.7

Let's discuss.

- Can we draw a line parallel to the X-axis at a distance of 6 units from it and below the X-axis ?
- Will all of the points (–3, –6), (10, –6), $(\frac{1}{2}, -6)$ be on that line ?
- What would be the equation of this line ?

Remember this !

If b > 0, and we draw the line $y = b$ through the point (0, b), it will be above the X-axis and parallel, to it. If b < 0, then the line $y = b$ will be below the X-axis and parallel to it.

The equation of a line parallel to the X-axis is in the form $y = b$.

Let's learn.

Lines parallel to the Y-axis

- On a graph paper, plot the following points.
  P(–4, 3), Q(–4, 0), R(–4, 1), S(–4, –2), T(–4, 2), U(–4, –3)

- Observe the co–ordinates of the points.
- Did you notice that the x co–ordinate of all the points are the same ?
- Are all the points collinear ?
- To which axis is this line parallel ?
- The x co–ordinate of every point on the line PS is -4. It is constant. Therefore, the line PS can be described by the equation $x = -4$. Every point whose x co–ordinate is -4 lies on the line PS. The equation of the line parallel to the Y–axis at a distance of 4 units and to the left of Y–axis is $x = -4$.

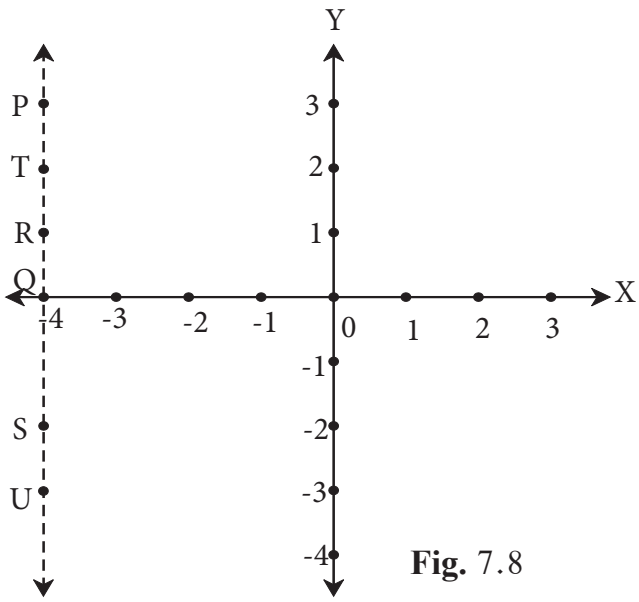


Fig. 7.8

Let's discuss.

- Can we draw a line parallel to the Y-axis at a distance of 2 units from it and to its right ?
- Will all of the points (2,10), (2,8), $(2, -\frac{1}{2})$ be on that line ?
- What would be the equation of this line ?

Remember this !

If we draw the line $x = a$ parallel to the Y-axis passing through the point $(a, 0)$ and if $a > 0$ then the line will be to the right of the Y-axis. If $a < 0$, then the line will be to the left of the Y-axis.

The equation of a line parallel to the Y-axis is in the form $x = a$.

Remember this !

(1) The y co-ordinate of every point on the X-axis is zero. Conversely, every point whose y co-ordinate is zero is on the X-axis. Therefore, the equation of the X axis is $y = 0$.

(2) The x co-ordinate of every point on the Y-axis is zero. Conversely, every point whose x co-ordinate is zero is on the Y-axis. Therefore, the equation of the Y-axis is $x = 0$.

Let's learn.

Graph of a linear equations

Ex. Draw the graphs of the equations $x = 2$ and $y = -3$.

Solution : (i) On a graph paper draw the X-axis and the Y-axis.

(ii) Since it is given that $x = 2$, draw a line on the right of the Y-axis at a distance of 2 units from it and parallel to it.

(iii) Since it is given that $y = -3$, draw a line below the X-axis at a distance of 3 units from it and parallel to it.

(iv) These lines, parallel to the two axes, are the graphs of the given equations.

(v) Write the co-ordinates of the point P, the point of intersection of these two lines.

(vi) Verify that the co-ordinates of the point P are (2, –3).

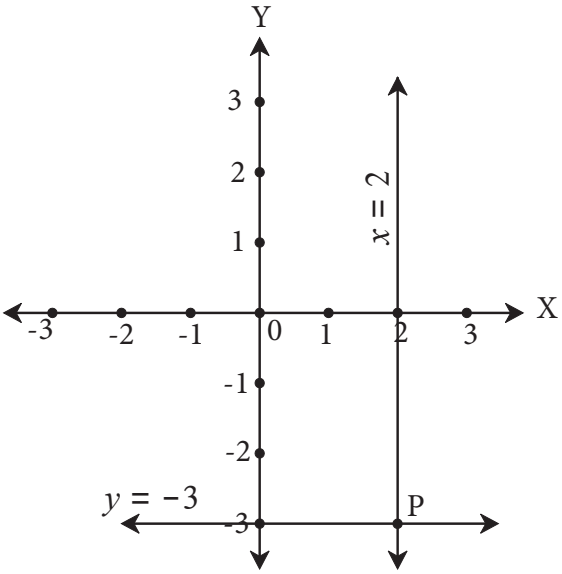


Fig. 7.9

The graph of a linear equation in the general form.

Activity : On a graph paper, plot the points (0,1) (1,3) (2,5). Are they collinear ? If so, draw the line that passes through them.

- Through which quadrants does this line pass ?
- Write the co-ordinates of the point at which it intersects the Y-axis.
- Show any point in the third quadrant which lies on this line. Write the co-ordinates of the point.

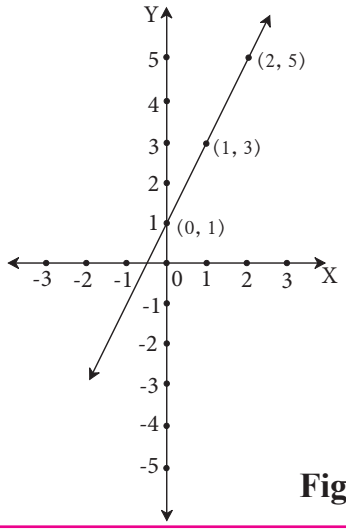


Fig. 7.10

Ex. $2x - y + 1 = 0$ is a linear equation in two variables in general form. Let us draw the graph of this equation.

Solution : $2x - y + 1 = 0$ means $y = 2x + 1$

Let us assume some values of x and find the corresponding values of y.

For example, If $x = 0$, then substituting this value of x in the equation we get $y = 1$.

Similarly, let us find the values of y when 0, 1, 2, $\frac{1}{2}$, -2 are some values of x and write these values in the table below in the form of ordered pairs.

x	0	1	2	$\frac{1}{2}$	-2
y	1	3	5	2	-3
(x, y)	(0,1)	(1,3)	(2,5)	$(\frac{1}{2}, 2)$	$(-2, -3)$

Now, let us plot these points. Let us verify that these points are collinear. Let us draw that line. The line is the graph of the equaiton $2x - y + 1 = 0$.

ICT Tools or Links

Use the Software Geogebra to draw the X and Y-axis. Plot several points. Find and study the co-ordinates of the points in 'Algebraic view'. Read the equations of lines that are parallel to the axes. Use the 'move' option to vary the positions of the lines. What are the equations of the X-axis and the Y-axis ?

Practice set 7.2

1. On a graph paper plot the points A (3,0), B(3,3), C(0,3). Join A, B and B, C. What is the figure formed ?
2. Write the equation of the line parallel to the Y-axis at a distance of 7 units from it to its left.
3. Write the equation of the line parallel to the X-axis at a distance of 5 units from it and below the X-axis.
4. The point Q(-3, -2) lies on a line parallel to the Y-axis. Write the equation of the line and draw its graph.

5. X-axis and line $x = -4$ are parallel lines. What is the distance between them?

6. Which of the equations given below have graphs parallel to the X-axis, and which ones have graphs parallel to the Y-axis ?

   (i) $x = 3$ (ii) $y - 2 = 0$ (iii) $x + 6 = 0$ (iv) $y = -5$

7. On a graph paper, plot the points A(2, 3), B(6, –1) and C(0, 5). If those points are collinear then draw the line which includes them. Write the co-ordinates of the points at which the line intersects the X-axis and the Y-axis.

8. Draw the graphs of the following equations on the same system of co-ordinates. Write the co-ordinates of their points of intersection.

   $x + 4 = 0$, $y - 1 = 0$, $2x + 3 = 0$, $3y - 15 = 0$

9. Draw the graphs of the equations given below.

   (i) $x + y = 2$ (ii) $3x - y = 0$ (iii) $2x + y = 1$

Problem set 7

1. Choose the correct alternative answer for the following quesitons.

   (i) What is the form of co-ordinates of a point on the X-axis ?

   (A) (b, b) (B) $(0, b)$ (C) $(a, 0)$ (D) (a, a)

   (ii) Any point on the line $y = x$ is of the form .....

   (A) (a, a) (B) $(0, a)$ (C) $(a, 0)$ (D) $(a, -a)$

   (iii) What is the equation of the X-axis ?

   (A) $x = 0$ (B) $y = 0$ (C) $x + y = 0$ (D) $x = y$

   (iv) In which quadrant does the point (–4, –3) lie ?

   (A) First (B) Second (C) Third (D) Fourth

   (v) What is the nature of the line which includes the points (–5,5), (6,5), (–3,5), (0,5) ?

   (A) Passes through the origin (B) Parallel to Y–axis

   (C) Parallel to X–axis (D) None of these

   (vi) Which of the points P (–1,1), Q (3,–4), R(1,–1), S (–2,–3), T (–4,4) lie in the fourth quadrant ?

   (A) P and T (B) Q and R (C) only S (D) P and R

2. Some points are shown in the figure 7.11.

With the help of it answer the following questions :

(i) Write the co-ordinates of the points Q and R.

(ii) Write the co-ordinates of the points T and M.

(iii) Which point lies in the third quadrant ?

(iv) Which are the points whose x and y co-ordinates are equal ?

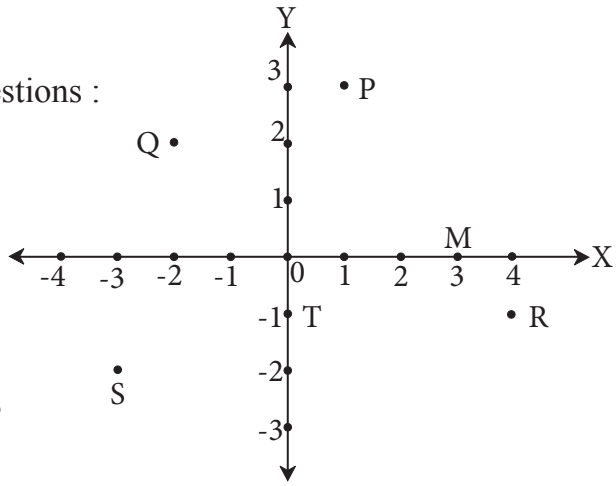


Fig.7.11

3. Without plotting the points on a graph, state in which quadrant or on which axis do the following point lie.

(i) (5, –3) (ii) (–7, –12) (iii) (–23, 4)

(iv) (–9, 5) (v) (0, –3) (vi) (–6, 0)

4. Plot the following points on the one and the same co-ordinate system.
A(1, 3), B(–3, –1), C(1, –4),
D(–2, 3), E(0, –8), F(1, 0)

5. In the graph alongside, line LM is parallel to the Y-axis. (Fig. 7.12)

(i) What is the distance of line LM from the Y-axis ?

(ii) Write the co-ordinates of the points P, Q and R.

(iii) What is the difference between the x co-ordinates of the points L and M?

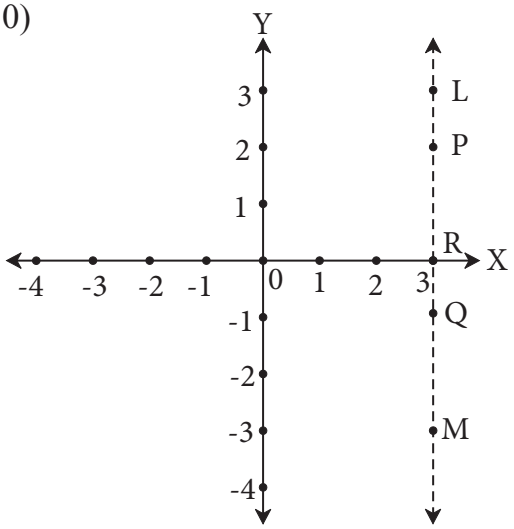


Fig.7.12

6. How many lines are there which are parallel to X-axis and having a distance 5 units?

7*. If 'a' is a real number, what is the distance between the Y-axis and the line $x = a$?