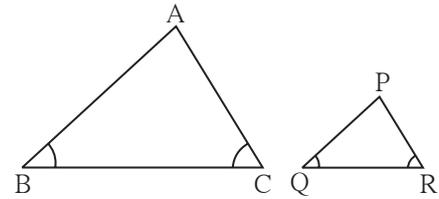




- If  $\Delta ABC \sim \Delta PQR$  then their corresponding sides are in the same proportions.

$$\text{So } \frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR}$$



**Fig. 8.2**

Let us see how to find the height of a tall tree using properties of similar triangles.

**Activity :** This experiment can be conducted on a clear sunny day. Look at the figure given alongside.

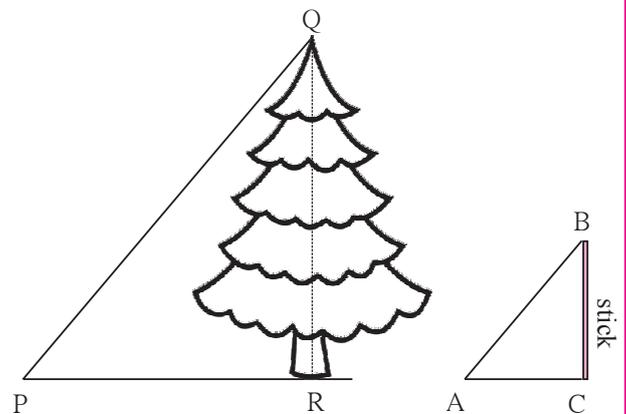
Height of the tree is QR, height of the stick is BC.

Thrust a stick in the ground as shown in the figure. Measure its height and length of its shadow. Also measure the length of the shadow of the tree. Rays of sunlight are parallel. So  $\Delta PQR$  and  $\Delta ABC$  are equiangular, means similar triangles. Sides of similar triangles are proportional.

$$\text{So we get } \frac{QR}{PR} = \frac{BC}{AC}.$$

Therefore, we get an equation,

$$\text{height of the tree} = QR = \frac{BC}{AC} \times PR$$



**Fig.8.3**

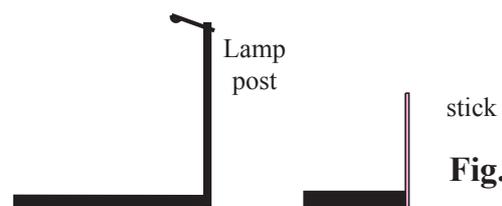
We know the values of PR, BC and AC. Substituting these values in this equation, we get length of QR, means height of the tree.



**Use your brain power !**

It is convenient to do this experiment between 11:30 am and 1:30 pm instead of doing it in the morning at 8'O clock. Can you tell why ?

**Activity :** You can conduct this activity and find the height of a tall tree in your surrounding. If there is no tree in the premises then find the height of a pole.



**Fig. 8.4**



## Let's learn.

### Terms related to right angled triangle

In right angled  $\Delta ABC$ ,  $\angle B = 90^\circ$ ,  $\angle A$  and  $\angle C$  are acute angles.

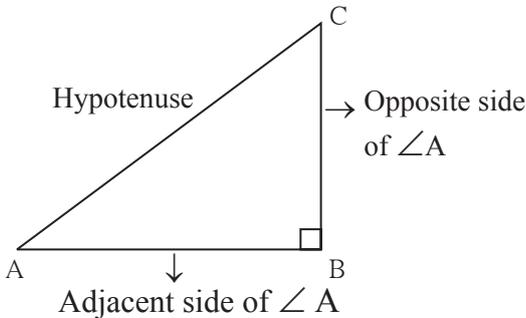


Fig. 8.5

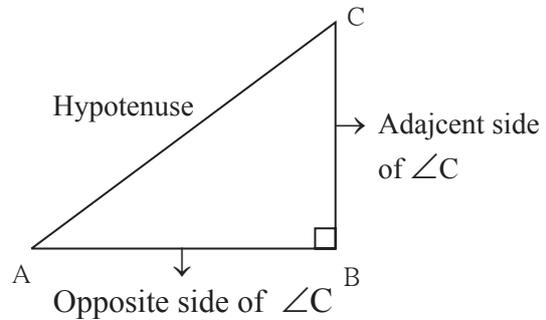


Fig. 8.6

**Ex.** In the figure 8.7,  $\Delta PQR$  is a right angled triangle. Write-

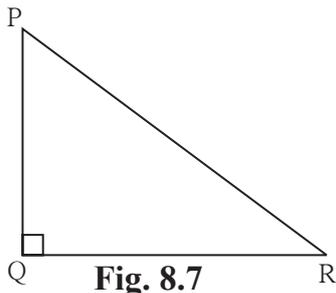


Fig. 8.7

- side opposite to  $\angle P = \dots$
- side opposite to  $\angle R = \dots$
- side adjacent to  $\angle P = \dots$
- side adjacent to  $\angle R = \dots$

### Trigonometric ratios

In the adjacent Fig.8.8 some right angled triangles are shown.  $\angle B$  is their common angle. So all right angled triangles are similar.

$$\Delta PQB \sim \Delta ACB$$

$$\therefore \frac{PB}{AB} = \frac{PQ}{AC} = \frac{BQ}{BC}$$

$$\therefore \frac{PQ}{AC} = \frac{PB}{AB} \quad \therefore \frac{PQ}{PB} = \frac{AC}{AB} \quad \dots \text{alternando}$$

$$\frac{QB}{BC} = \frac{PB}{AB} \quad \therefore \frac{QB}{PB} = \frac{BC}{AB} \quad \dots \text{alternando}$$

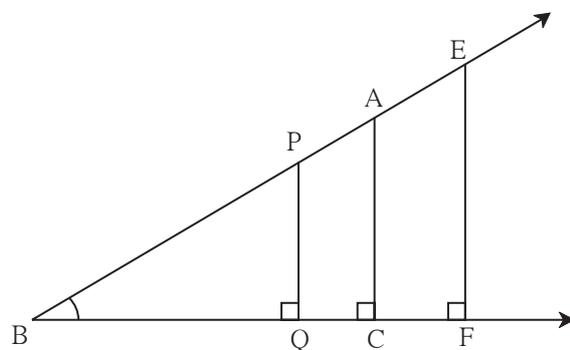


Fig. 8.8



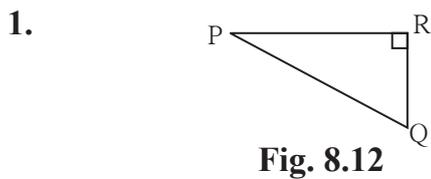
$$\sin C = \sin \theta = \frac{AB}{AC}, \quad \cos C = \cos \theta = \frac{BC}{AC}, \quad \tan C = \tan \theta = \frac{AB}{BC}$$



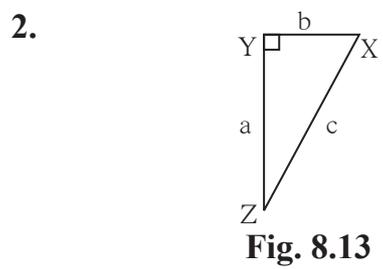
**Remember this !**

- $\sin \text{ ratio} = \frac{\text{opposite side}}{\text{hypotenuse}}$
- $\cos \text{ ratio} = \frac{\text{adjacent side}}{\text{hypotenuse}}$
- $\tan \text{ ratio} = \frac{\text{opposite side}}{\text{adjacent side}}$
- $\sin \theta = \frac{\text{opposite side of } \angle \theta}{\text{hypotenuse}}$
- $\cos \theta = \frac{\text{adjacent side of } \angle \theta}{\text{hypotenuse}}$
- $\tan \theta = \frac{\text{opposite side of } \angle \theta}{\text{adjacent side of } \angle \theta}$

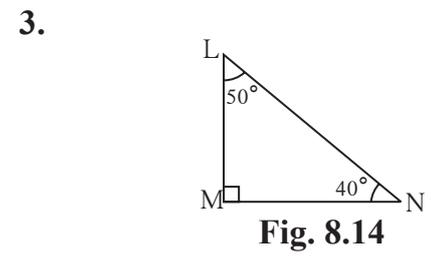
**Practice set 8.1**



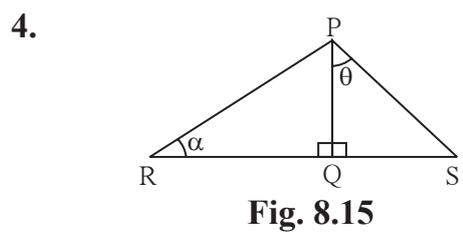
In the Fig.8.12,  $\angle R$  is the right angle of  $\Delta PQR$ . Write the following ratios.  
 (i)  $\sin P$  (ii)  $\cos Q$  (iii)  $\tan P$  (iv)  $\tan Q$



In the right angled  $\Delta XYZ$ ,  $\angle XYZ = 90^\circ$  and  $a, b, c$  are the lengths of the sides as shown in the figure. Write the following ratios,  
 (i)  $\sin X$  (ii)  $\tan Z$  (iii)  $\cos X$  (iv)  $\tan X$ .



In right angled  $\Delta LMN$ ,  $\angle LMN = 90^\circ$   
 $\angle L = 50^\circ$  and  $\angle N = 40^\circ$ ,  
 write the following ratios.  
 (i)  $\sin 50^\circ$  (ii)  $\cos 50^\circ$   
 (iii)  $\tan 40^\circ$  (iv)  $\cos 40^\circ$



In the figure 8.15,  $\angle PQR = 90^\circ$ ,  
 $\angle PQS = 90^\circ$ ,  $\angle PRQ = \alpha$  and  $\angle QPS = \theta$   
 Write the following trigonometric ratios.  
 (i)  $\sin \alpha$ ,  $\cos \alpha$ ,  $\tan \alpha$   
 (ii)  $\sin \theta$ ,  $\cos \theta$ ,  $\tan \theta$



**\* For more information**

$$\frac{1}{\sin \theta} = \operatorname{cosec} \theta, \quad \frac{1}{\cos \theta} = \sec \theta, \quad \frac{1}{\tan \theta} = \cot \theta$$

It means cosec  $\theta$ , sec  $\theta$  and cot  $\theta$  are inverse ratios of sin  $\theta$ , cos  $\theta$  and tan  $\theta$  respectively.

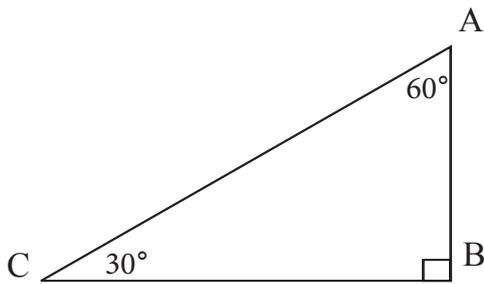
- sec  $\theta = \operatorname{cosec} (90 - \theta)$
- cosec  $\theta = \sec (90 - \theta)$
- tan  $\theta = \cot (90 - \theta)$
- cot  $\theta = \tan (90 - \theta)$



**Let's recall.**

**Theorem of 30°- 60°-90° triangle :**

We know that if the measures of angles of a triangle are 30°, 60°, 90° then side opposite to 30° angle is half of the hypotenuse and side opposite to 60° angle is  $\frac{\sqrt{3}}{2}$  of hypotenuse.



**Fig. 8.17**

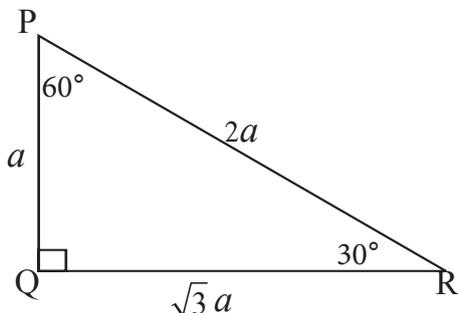
In the Fig. 8.17,  $\Delta ABC$  is a right angled triangle.  $\angle C = 30^\circ$ ,  $\angle A = 60^\circ$ ,  $\angle B = 90^\circ$ .

$$\therefore AB = \frac{1}{2} AC \text{ and } BC = \frac{\sqrt{3}}{2} AC$$



**Let's learn.**

**Trigonometric ratios of 30° and 60° angles**



**Fig. 8.18**

In right angled  $\Delta PQR$  if  $\angle R = 30^\circ$ ,  $\angle P = 60^\circ$ ,  $\angle Q = 90^\circ$  and  $PQ = a$

$$\text{then } PQ = \frac{1}{2} PR$$

$$a = \frac{1}{2} PR$$

$$\therefore PR = 2a$$

$$QR = \frac{\sqrt{3}}{2} PR$$

$$QR = \frac{\sqrt{3}}{2} \times 2a$$

$$QR = \sqrt{3} a$$

$\therefore$  If  $PQ = a$ , then  $PR = 2a$  and  $QR = \sqrt{3} a$

(I) Trigonometric ratios of the  $30^\circ$  angle

$$\sin 30^\circ = \frac{PQ}{PR} = \frac{a}{2a} = \frac{1}{2}$$

$$\cos 30^\circ = \frac{QR}{PR} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$$

$$\tan 30^\circ = \frac{PQ}{QR} = \frac{a}{\sqrt{3}a} = \frac{1}{\sqrt{3}}$$

(II) Trigonometric ratios of  $60^\circ$  angle

$$\sin 60^\circ = \frac{QR}{PR} = \frac{\sqrt{3}a}{2a} = \frac{\sqrt{3}}{2}$$

$$\cos 60^\circ = \frac{PQ}{PR} = \frac{a}{2a} = \frac{1}{2}$$

$$\tan 60^\circ = \frac{QR}{PQ} = \frac{\sqrt{3}a}{a} = \sqrt{3}$$

In right angled  $\Delta PQR$ ,  $\angle Q = 90^\circ$ . Therefore  $\angle P$  and  $\angle R$  are complimentary angles of each other. Verify the relation between sine and cosine ratios of complimentary angles here also.

$$\sin \theta = \cos (90 - \theta)$$

$$\sin 30^\circ = \cos (90^\circ - 30^\circ) = \cos 60^\circ$$

$$\sin 30^\circ = \cos 60^\circ$$

$$\cos \theta = \sin (90 - \theta)$$

$$\cos 30^\circ = \sin (90^\circ - 30^\circ) = \sin 60^\circ$$

$$\cos 30^\circ = \sin 60^\circ$$



**Remember this !**

$\sin 30^\circ = \frac{1}{2}$	$\cos 30^\circ = \frac{\sqrt{3}}{2}$	$\tan 30^\circ = \frac{1}{\sqrt{3}}$
$\sin 60^\circ = \frac{\sqrt{3}}{2}$	$\cos 60^\circ = \frac{1}{2}$	$\tan 60^\circ = \sqrt{3}$

(III) Trigonometric ratios of the  $45^\circ$  angle

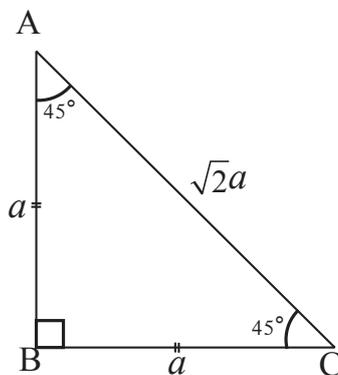


Fig.8.19

In right angled  $\Delta ABC$ ,  $\angle B = 90^\circ$ ,  $\angle A = 45^\circ$ ,  $\angle C = 45^\circ \therefore$  This is an isosceles triangle.

Suppose  $AB = a$  then  $BC = a$ .

Using Pythagoras' theorem, let us find the length of  $AC$ .

$$AC^2 = AB^2 + BC^2$$

$$= a^2 + a^2$$

$$AC^2 = 2a^2$$

$$\therefore AC = \sqrt{2}a$$

In the Fig. 8.19,  $\angle C = 45^\circ$

$$\sin 45^\circ = \frac{AB}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$$

$$\cos 45^\circ = \frac{BC}{AC} = \frac{a}{\sqrt{2}a} = \frac{1}{\sqrt{2}}$$

$$\tan 45^\circ = \frac{AB}{BC} = \frac{a}{a} = 1$$



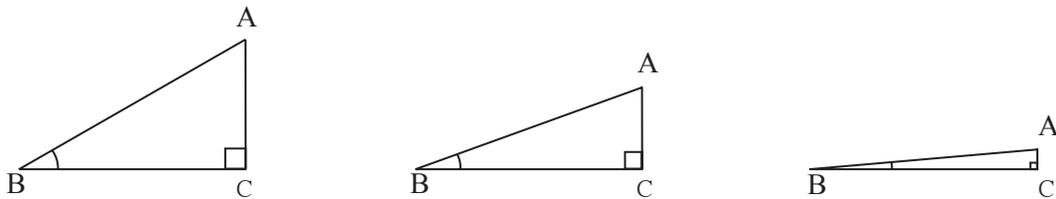
**Remember this !**

$$\sin 45^\circ = \frac{1}{\sqrt{2}},$$

$$\cos 45^\circ = \frac{1}{\sqrt{2}},$$

$$\tan 45^\circ = 1$$

(IV) Trigonometric ratios of the angle  $0^\circ$  and  $90^\circ$

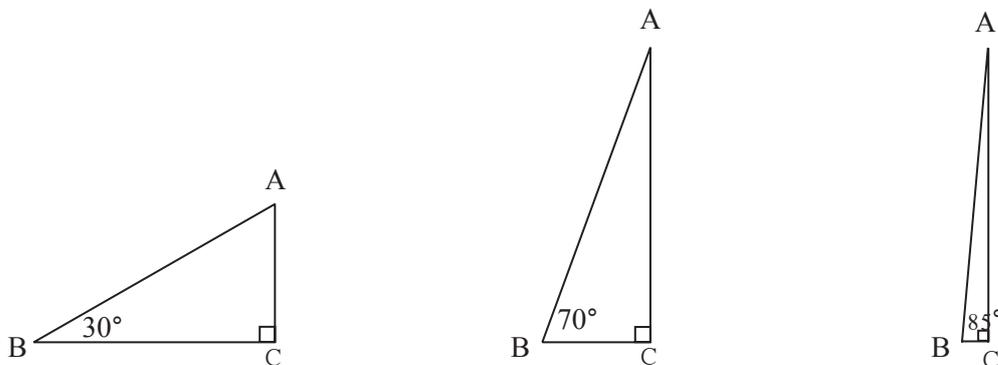


**Fig.8.20**

In the right angled  $\Delta ACB$ ,  $\angle C = 90^\circ$  and  $\angle B = 30^\circ$ . We know that  $\sin 30^\circ = \frac{AC}{AB}$ . Keeping the length of side AB constant, if the measure of  $\angle B$  goes on decreasing the length of AC, which is opposite to  $\angle B$  also goes on decreasing. So as the measure of  $\angle B$  decreases, then value of  $\sin \theta$  also decreases.

$\therefore$  when measure of  $\angle B$  becomes  $0^\circ$ , then length of AC becomes 0.

$$\therefore \sin 0^\circ = \frac{AC}{AB} = \frac{0}{AB} = 0 \quad \therefore \sin 0^\circ = 0$$



**Fig.8.21**



**Solved Examples :**

**Ex. (1)** Find the value of  $2\tan 45^\circ + \cos 30^\circ - \sin 60^\circ$

**Solution :**

$$2\tan 45^\circ + \cos 30^\circ - \sin 60^\circ$$

$$= 2 \times 1 + \frac{\sqrt{3}}{2} - \frac{\sqrt{3}}{2}$$

$$= 2 + 0$$

$$= 2$$

**Ex. (2)** Find the value of  $\frac{\cos 56^\circ}{\sin 34^\circ}$

**Solution :**  $56^\circ + 34^\circ = 90^\circ$  means 56 and 34 are the measures of complimentary angles.

$$\sin \theta = \cos (90 - \theta)$$

$$\therefore \sin 34^\circ = \cos (90 - 34)^\circ = \cos 56^\circ$$

$$\therefore \frac{\cos 56^\circ}{\sin 34^\circ} = \frac{\cos 56^\circ}{\cos 56^\circ} = 1$$

**Ex. 3** In right angled  $\Delta ACB$ , If  $\angle C = 90^\circ$ ,  $AC = 3$ ,  $BC = 4$ .

Find the ratios  $\sin A$ ,  $\sin B$ ,  $\cos A$ ,  $\tan B$

**Solution :** In right angled  $\Delta ACB$ , using Pythagoras' theorem,

$$AB^2 = AC^2 + BC^2$$

$$= 3^2 + 4^2 = 5^2$$

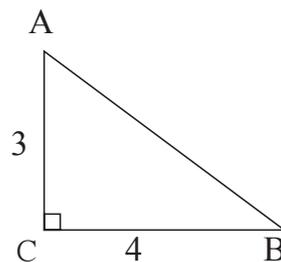
$$\therefore AB = 5$$

$$\sin A = \frac{BC}{AB} = \frac{4}{5}$$

$$\cos A = \frac{AC}{AB} = \frac{3}{5}$$

$$\text{and } \sin B = \frac{AC}{AB} = \frac{3}{5}$$

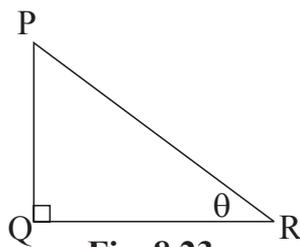
$$\tan B = \frac{AC}{BC} = \frac{3}{4}$$



**Fig. 8.22**

**Ex. 4** In right angled triangle  $\Delta PQR$ ,  $\angle Q = 90^\circ$ ,  $\angle R = \theta$  and if  $\sin \theta = \frac{5}{13}$  then find  $\cos \theta$  and  $\tan \theta$ .

**Solution :**



**Fig. 8.23**

In right angled  $\Delta PQR$ ,  $\angle R = \theta$

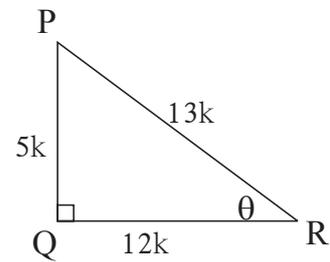
$$\sin \theta = \frac{5}{13}$$

$$\therefore \frac{PQ}{PR} = \frac{5}{13}$$

∴ Let  $PQ = 5k$  and  $PR = 13k$

Let us find  $QR$  by using Pythagoras' theorem,

$$\begin{aligned} PQ^2 + QR^2 &= PR^2 \\ (5k)^2 + QR^2 &= (13k)^2 \\ 25k^2 + QR^2 &= 169k^2 \\ QR^2 &= 169k^2 - 25k^2 \\ QR^2 &= 144k^2 \\ QR &= 12k \end{aligned}$$



**Fig. 8.24**

Now, in right angled  $\Delta PQR$ ,  $PQ = 5k$ ,  $PR = 13k$  and  $QR = 12k$

$$\therefore \cos \theta = \frac{QR}{PR} = \frac{12k}{13k} = \frac{12}{13}, \tan \theta = \frac{PQ}{QR} = \frac{5k}{12k} = \frac{5}{12}$$



**Use your brain power!**

- (1) While solving above example, why the lengths of  $PQ$  and  $PR$  are taken  $5k$  and  $13k$  ?
- (2) Can we take the lengths of  $PQ$  and  $PR$  as  $5$  and  $13$  ? If so then what changes are needed in the writing of the solution.

**Important Equation in Trigonometry**

$\Delta PQR$  is a right angled triangle.

$$\angle PQR = 90^\circ, \angle R = \theta$$

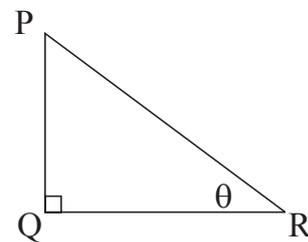
$$\sin \theta = \frac{PQ}{PR} \dots\dots\dots(I)$$

$$\text{and } \cos \theta = \frac{QR}{PR} \dots\dots\dots(II)$$

Using Pythagoras' theorem,

$$PQ^2 + QR^2 = PR^2$$

$$\therefore \frac{PQ^2}{PR^2} + \frac{QR^2}{PR^2} = \frac{PR^2}{PR^2} \dots \text{dividing each term by } PR^2$$



**Fig.8.25**

$$\therefore \left(\frac{PQ}{PR}\right)^2 + \left(\frac{QR}{PR}\right)^2 = 1$$

$$\therefore (\sin \theta)^2 + (\cos \theta)^2 = 1 \text{ ..from (I) \& (II)}$$



## Problem set 8

1. Choose the correct alternative answer for following multiple choice questions.

(i) Which of the following statements is true ?

(A)  $\sin \theta = \cos (90 - \theta)$                       (B)  $\cos \theta = \tan (90 - \theta)$

(C)  $\sin \theta = \tan (90 - \theta)$                       (D)  $\tan \theta = \tan (90 - \theta)$

(ii) Which of the following is the value of  $\sin 90^\circ$  ?

(A)  $\frac{\sqrt{3}}{2}$               (B) 0              (C)  $\frac{1}{2}$               (D) 1

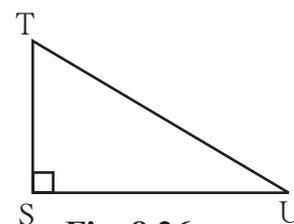
(iii)  $2 \tan 45^\circ + \cos 45^\circ - \sin 45^\circ = ?$

(A) 0              (B) 1              (C) 2              (D) 3

(iv)  $\frac{\cos 28^\circ}{\sin 62^\circ} = ?$

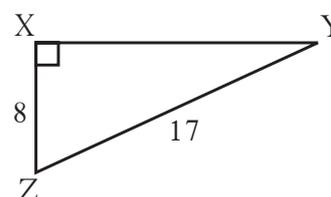
(A) 2              (B) -1              (C) 0              (D) 1

2. In right angled  $\Delta$  TSU,  $TS = 5$ ,  $\angle S = 90^\circ$ ,  
 $SU = 12$  then find  $\sin T$ ,  $\cos T$ ,  $\tan T$ .  
 Similarly find  $\sin U$ ,  $\cos U$ ,  $\tan U$ .



**Fig. 8.26**

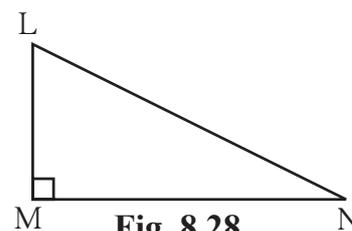
3. In right angled  $\Delta$  YXZ,  $\angle X = 90^\circ$ ,  $XZ = 8$  cm,  
 $YZ = 17$  cm, find  $\sin Y$ ,  $\cos Y$ ,  $\tan Y$ ,  
 $\sin Z$ ,  $\cos Z$ ,  $\tan Z$ .



**Fig. 8.27**

4. In right angled  $\Delta$  LMN, if  $\angle N = \theta$ ,  $\angle M = 90^\circ$ ,  
 $\cos \theta = \frac{24}{25}$ , find  $\sin \theta$  and  $\tan \theta$

Similarly, find  $(\sin^2 \theta)$  and  $(\cos^2 \theta)$ .



**Fig. 8.28**

5. Fill in the blanks.

(i)  $\sin 20^\circ = \cos \square^\circ$

(ii)  $\tan 30^\circ \times \tan \square^\circ = 1$

(iii)  $\cos 40^\circ = \sin \square^\circ$

